Math 2423 homework

10. (due 9/7) In class we class calculated the area under $y = x^3$ by approximating it by rectangles. Adapt this method to calculate the area of the triangle T bounded by the x-axis, the line $x = b$, and the graph of $y = mx$ where $m > 0$. Specifically:

    1. Partition the interval $[0, b]$ into n equal subintervals and calculate the total area U_n of the rectangles with heights $m(b/n)$, $m(2b/n)\ldots$, $m(nb/n)$ (the "outside" ones whose union contains T). Simplify the sum using the formula $\sum_{i=1}^{n} i = n(n+1)/2$.
    2. Calculate the total area L_n of the rectangles with heights 0, $m(b/n)\ldots$, $m((n-1)b/n)$ (the "inside" ones whose union is contained in T), and simplify.
    3. Your drawings for the previous steps should show that $L_n < A < U_n$, where A is the area of T. Calculate that $\lim_{n\to\infty} L_n = \lim_{n\to\infty} U_n = mb^2/2$, to deduce that A is also $mb^2/2$.

11. (9/7) 5.1 # 4, 20, 21. (Explain 20 and 21 with graphs, of course.)

12. (9/7) Estimate the number 2 as follows.

    1. Partition the interval $[0, 4]$ into four equal-length subintervals.
    2. Using this partition and right-hand endpoints, calculate a Riemann sum to estimate $f(4) - f(0)$ for the function $f(x) = \sqrt{x}$.
    3. Explain geometrically why this Riemann sum is an underestimate.

13. (9/12) In this problem, $f(x) = x^2$, the interval is $[-1, 4]$, and the partition is $-1 < 0 < 1 < 2 < 4$.

    (a) What is the mesh of this partition?

    (b) Calculate the Riemann sum that uses the midpoints of the subintervals as the x_i^*.

    (c) Find the largest Riemann sum.

    (d) Find the smallest Riemann sum.

14. (9/12) 5.2 # 35-41, 43.