Math 2423 homework

15. (9/9) I wrote the wrong thing in a couple of places when I was explaining the Fundamental Theorem of Calculus in Wednesday's lecture (sorry about that!), so let's try to clear up any confusion created.

    (a) Draw our picture of the argument, decomposing the region under f and between a and $x+h$, whose total area is $\int_a^{x+h} f(t)\,dt$, into three regions: (1) the area under f between a and x, whose area is $\int_a^x f(t)\,dt$, (2) a rectangle of area $f(x)h$, and (3) a small almost triangular region, whose base has length h and whose top curved side is the graph of f between x and $x+h$.

    (b) Observe that there is a corresponding formula

    $$\int_a^{x+h} f(t)\,dt = \int_a^x f(t)\,dt + f(x)\,h + E(h)\ ,$$

    where $E(h)$ is the area of the almost triangular region.

    (c) Draw a larger picture of the almost triangular region, and observe that its area is approximately $\frac{1}{2}f'(x)h^2$.

    (d) Observe that this indicates that $\lim_{h\to 0}\dfrac{E(h)}{h} = 0$. Understand why this shows that

    $$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)\ .$$

    (e) Use this picture, in the future, as part of your intuition for the rate of change: the rectangle is the linear part of the change of area, and the almost triangular region, whose area is "quadratic in h", is the error of linear approximation.

    You don't need to turn anything in, but working through these steps will help you assimilate several important ideas.

16. (9/14) As many of 5.3 # 19-35 as needed. They all use the FTC, many of them using the fact that $\dfrac{x^{n+1}}{n+1}$ is an antiderivative of x^n (in particular, cx is an antiderivative of a constant function c), provided that $n \neq -1$. For example, 5.3 # 28 is $\int_0^1 3 + x\sqrt{x}\,dx = \int_0^1 3\,dx + \int_0^1 x^{3/2}\,dx = 3\cdot 1 - 3\cdot 0 + \dfrac{1^{5/2}}{5/2} - \dfrac{0^{5/2}}{5/2} = 3 - 0 + \dfrac{2}{5} - 0 = \dfrac{17}{5}$, and 5.3 # 32 is $\int_0^{\pi/4} \sec(\theta)\tan(\theta)\,d\theta = \sec(\pi/4) - \sec(0) = \sqrt{2} - 1$. Turn in 5.3 # 24, 26, 29, 31, 34-36.