Mathematics 2423-001 Name (please print) ______________________

Exam I

September 19, 2011

Instructions: Give concise answers, but clearly indicate your reasoning. It is *not* expected that you will be able to answer all the questions, just do whatever you can in 50 minutes.

I. For the function $f(x) = x^2$ on the interval $[-1, 6]$ and the partition $-1 < 2 < 4 < 6$ of $[-1, 6]$:
(6)

(a) Calculate the Riemann sum that uses the midpoints of the subintervals as the x_i^*.

(b) Calculate the smallest Riemann sum.

II. (a) State the definition we used for the *rate of change* of a function f at a point a (in the definition, use m for the rate of change).
(9)

(b) Use our definition to verify that the rate of change of $x^3 + 8x$ at 0 is 8. That is, taking this function as $f(x)$, $a = 0$, and 8 as the rate of change, write $f(0+h) = h^3 + 8h$ as an expression involving the rate of change and the error $E(h)$, then analyze $E(h)$ to verify that 8 is the correct rate of change.

(c) Use our definition to verify that the rate of change of $x^3 + 8x$ at 0 is not -2.

III. Without worrying about the hypotheses, state both parts of the Fundamental Theorem of Caclulus.
(6)

IV. Use the Mean Value Theorem to verify that if x and y are any two numbers greater than 1, then
(5) $|\sqrt{x} - \sqrt{y}| \leq |x - y|/2$.

V. For the function $f(x) = x^2$, write the Riemann sum of the form $\sum_{i=1}^{n} f(x_i)\Delta x$ for the partition of the interval
(8)
$[0, 2]$ into n equal intervals, where $x_i = 2i/n$. Use the formula $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}$ to rewrite this Riemann sum as an expression with only one term, then take its limit to find the area under f on $[0, 2]$.

VI. Write an expression, involving an integral, for a function $f(x)$ such that $f(2) = 0$ and f is an antiderivative
(3) of $B_2(x)$ ($B_2(x)$ is a certain continuous function defined on the entire real line, called the *Bessel function of the second kind.*)

VII. Calculate the following.
(16)

(a) $\displaystyle\int_0^1 (\sqrt{x} + 1)^2 \, dx$

(b) $\displaystyle\int_0^{3\pi/2} \sin(y) \, dy$

(c) $\displaystyle\lim_{\text{mesh}(\{x_0, x_1, x_2, \ldots, x_n\}) \to 0} \sum_{i=1}^{n} \sin(x_i^*) \, \Delta x_i$, where $\text{mesh}(\{x_0, x_1, x_2, \ldots, x_n\})$ is the mesh of the partition $0 = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = 3\pi/2$, $\Delta x_i = x_i - x_{i-1}$, and the limit is taken over all partitions and all choices of the sample points $x_i^* \in [x_{i-1}, x_i]$.

(d) The number $g(1)$, where $\dfrac{dg}{dx} = \dfrac{d}{dx}\left(\dfrac{(x^2+1)(x-1)}{(x^4+1)^3}\right)$ and $g(0) = 4$.