

Exam III

November 16, 2011

Instructions: Give concise answers, but clearly indicate your reasoning. It is *not* expected that you will be able to answer all the questions, just do whatever you can in 50 minutes.

I. Find the point on the graph of $y = \cosh(x)$ where the tangent line has slope 2. Express the answer as an (4) expression involving the logarithm function, not an inverse hyperbolic trig function.

II. Evaluate the following integrals.

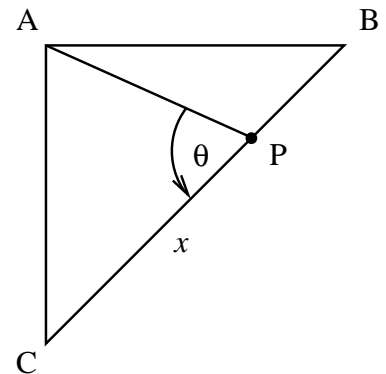
(16) 1. $\int \tan^3(x) dx$

2. $\int \frac{\log_{10}(x)}{x} dx$

3. $\int x^3 e^{-x^2} dx$. You may make use of the fact that $\int x e^{-x^2} dx = -e^{-x^2}/2 + C$.

4. $\int \frac{1}{\sqrt{x}(1+x)} dx$

III. The triangle ABC is an isosceles right triangle whose legs AB and AC each have length 2. Let $0 < x \leq 2\sqrt{2}$ be the distance from C to the point P on the hypotenuse of ABC . Express the angle θ as a function of x . (Hint: draw the horizontal line from P to AC .)



IV. Evaluate the following limits.

(12) 1. $\lim_{x \rightarrow \infty} x \tan(1/x)$

2. $\lim_{x \rightarrow 0} (1 - 2x)^{1/x}$

3. $\lim_{x \rightarrow 0} \frac{1}{x^3} \int_0^x \sin\left(\frac{1}{2}\pi t^2\right) dt$

V. Define what it means to say that a function f is *injective* (also called *one-to-one*). Assuming that f is an (4) injective function with domain A and range B , define what it means to say that a function g is the *inverse* of f .

VI. Obtain the reduction formula $\int (\ln(x))^n dx = x(\ln(x))^n - n \int (\ln(x))^{n-1} dx$. (4)

VII. Find the exact values of $\arctan(\tan(1))$ and $\arctan(\tan(6))$,

(3)

VIII. Calculate the following derivatives.

(6) 1. $\frac{d}{dx} \log_{10}(\ln(x))$

2. $\frac{d}{dx} 10^{\ln(x)}$