Mathematics 2934-010 Name (please print) ______________________

Quiz 6

December 1, 2011

Instructions: Give concise answers, but clearly indicate your reasoning. Remember that $\nabla f$ and grad($f$) both mean the gradient of $f$.

**I.** (2) State Green's Theorem for a domain $D$ with boundary a positively oriented simple loop $C$. Include any important conditions that the functions $P$ and $Q$ in its statement must satisfy.

**II.** (3) Let $C$ be the unit circle, oriented counterclockwise. Use Green's Theorem to calculate $\int ((2x + y^2)\,\vec{\imath} + (2xy - 2x + \sin(y^2))\,\vec{\jmath}) \cdot d\vec{r}$.

**III.** (5) Let $T$ be the triangle with vertices at $(1, 0)$, $(0, 1)$, and $(0, 0)$. Let $C$ be the boundary of $T$, with the positive orientation.

(a) Use Green's Theorem to verify that $\int_C x\,dy$ equals the area of $T$.

(b) Let $C_1$ be the side of $T$ from $(0, 1)$ to $(0, 0)$, $C_2$ be the side of $T$ from $(0, 0)$ to $(1, 0)$, and $C_3$ be the side of $T$ from $(1, 0)$ to $(0, 1)$. Without calculating them, explain why $\int_{C_1} x\,dy$ and $\int_{C_2} x\,dy$ are zero.

(c) Use the parameterization $x = 1-t$, $y = t$, $0 \leq t \leq 1$ to calculate $\int_{C_3} x\,dy$ (from parts (a) and (b), the answer should equal the area of $T$, that is, $\frac{1}{2}$).

**IV.** (3) (a) Define what it means to say that a domain $D$ in the plane is *simply-connected.*

(b) Tell the important and non-obvious property we have studied that is true for vector fields $P\vec{\imath} + Q\vec{\jmath}$ on simply-connected planar domains, but not necessarily true for domains that are not simply-connected.

(c) Give an example of a vector field illustrating that the property in (b) is not necessarily true for non-simply-connected domains. You do not need to verify the properties, just tell the vector field and the domain.

**V.** (3) Let $f$ be a scalar field in $\mathbb{R}^3$ (that is, a function of $(x, y, z)$) and let $\vec{F}$ be a vector field in $\mathbb{R}^3$. Determine whether each of the following is a scalar field, a vector field, or meaningless.

(a) $\text{div}(\text{curl}(\text{curl}(\vec{F})))$

(b) $\text{div}(\vec{F} \times \text{curl}(\vec{F}))\ \text{grad}(f)$

(c) $\text{curl}(\vec{F}) \cdot (\text{curl}(\vec{F}) \times \text{curl}(\vec{F}))$

**VI.** (4) Let $C$ be an oriented path in the plane, with unit tangent $\vec{T}$, and let $f$ be a differentiable function in the plane.

(a) What is a simple interpretation of $\nabla f \cdot \vec{T}$?

(b) How is $\nabla f \cdot \vec{T}$ related to $\int_C \nabla f \cdot d\vec{r}$?

(c) What do (a) and (b) tell us, at least intuitively, about $\int_C \nabla f \cdot d\vec{r}$?