Mathematics 2934-010 Name (please print) ____________________

Exam II

October 17, 2011

Instructions: Give concise answers, but clearly indicate your reasoning. Do not simplify the answers unless you find yourself with enough time at the end.

I. Calculate the following.
(12)

1. $\dfrac{\partial}{\partial y}\left(\displaystyle\int_y^{x^2} e^{-t^2}\,dt\right)$

2. $\dfrac{\partial}{\partial x}\left(\displaystyle\int_y^{x^2} e^{-t^2}\,dt\right)$

3. $f_{tx}(x,y,z,t)$ if $f_x(x,y,z,t) = \dfrac{1}{x^2y^2z^2t^2}$

4. df if $f(x,y,z) = x^3yz$

II. Clairaut's Theorem tells us there is no function $g(x,y)$ for which $\nabla g = 2x\sin(y)\vec{\imath} + ye^x\vec{\jmath}$. Explain why.
(4)

III. Let $f(x,y,z) = x^2 + y^2 + z^3$.
(8)

1. Find the maximum directional rate of change at $(x,y,z) = (-2,0,1)$.

2. Find the rate of change at $(x,y,z) = (-1,-1,-2)$ in the direction $2\vec{\imath} - \vec{\jmath} - 2\vec{k}$.

3. Write an equation for the tangent plane to the level surface through $(-2,0,1)$.

IV. Let $f(x,y)$ be a differentiable function of x and y.
(7)

(a) Use the Chain Rule to calculate $\dfrac{\partial f}{\partial \theta}$, where θ is the polar angle. Express $\dfrac{\partial f}{\partial \theta}$ purely in terms of x, y, $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ (that is, without using r or θ).

(b) In the case of $f(x,y) = x^2 + y^2$, the formula should give $\dfrac{\partial f}{\partial \theta} = 0$. Why is the fact that $\dfrac{\partial}{\partial \theta}(x^2+y^2) = 0$ geometrically obvious without doing any calculations at all?

V. (a) Define the *graph* of a function $f(x,y)$ of two variables.
(5)

(b) For a certain function $f(x,y)$, $f(1,4) = 5$ and $\dfrac{\partial f}{\partial y}(1,4) = 3$. Let L be the line tangent to the graph of f at the point $(1,4,5)$ and lying in the plane $x = 1$. Tell a direction vector for L.

VI. Suppose that $\vec{T}(t)$ is the unit tangent vector to some curve. Show that $\vec{T}'(t)$ is orthogonal (that is, perpendicular) to $\vec{T}(t)$.
(3)

VII. For the vector-valued function $\vec{r}(t) = 4\cos(t/2)\vec{\imath} + 4\sin(t/2)\vec{\jmath}$, the velocity is $\vec{v}(t) = -2\sin(t/2)\vec{\imath} + 2\cos(t/2)\vec{\jmath}$.
(10)

(a) Draw a large graph showing the curve traced out by $\vec{r}(t)$ for $0 \le t \le 3\pi$.

(b) Calculate the velocity and speed, and draw the velocity vectors for $t = \pi$ and $t = 1$, showing with reasonable accuracy their locations and lengths.

(c) Calculate $\|\vec{T}'(t)\|/(ds/dt)$.

(d) Calculate the tangential component of the vectors $\vec{w}(t) = -3\vec{\imath}$.

VIII. A particle moves with acceleration $2t\,\vec{\imath} - 12t\,\vec{\jmath}$. At time $t = 0$, it is located at $(0, 1, 0)$ and is moving with
(4) velocity $\vec{k}$. Find its position function $\vec{r}(t)$.