Mathematics 2443-004

Examination I Form A

February 9, 2000

Name (please print) ______________________

Student Number ______________

I. Calculate the following partial derivatives.
(15)

1. f_v if $f(u,v) = \tan^{-1}(\frac{u}{v})$

2. $\dfrac{\partial R}{\partial R_2}$ if $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$

3. $\dfrac{\partial^2 w}{\partial r \partial \theta}$ if $w = \cos(r\theta)$

II. (5) On an xy-coordinate system, draw a rough map of Oklahoma with Norman at the origin $(0, 0)$. Assume that the wind is blowing from southwest to northeast. Let $P(x, y)$ be the air pressure function. Draw some level curves of P, and draw some of the gradient vectors ∇P.

III. (10) Calculate the following partial derivatives.

1. $\dfrac{\partial z}{\partial \theta}$ if $z = f(x, y)$ (where θ is the polar angle coordinate). Express the final answer without using r and θ.

2. $g'(h)$ if $f(x, y)$ is a function of x and y, (x_0, y_0) is a certain pint in the domain of f, and $g(h) = f(x_0 + ah, y_0 + bh)$

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IV. For the function $g(x, y) = e^x \cos(y)$, calculate the following.
(9)

1. The gradient ∇g.

2. The directional derivative of g at the point $(2, \pi/6)$ in the direction of the vector $2\vec{\imath} - \vec{\jmath}$.

3. The maximum rate of change of f at the point $(2, \pi/6)$ and the direction in which it occurs.

V. Show that $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{xy + yz + zx}{x^2 + y^2 + z^2}$ does not exist.
(4)

VI. (3) Suppose that $g(x, y)$ be a function of two variables. Define what it means to say that (a, b) is a *critical point* of g.

VII. (5) Consider the function $f(x, y) = 2x^3 + y^4$ with domain the unit disc D consisting of all points (x, y) with $x^2 + y^2 \leq 1$. Determine all critical points of f. Write an expression for the values of f on the *boundary* of D as a function of a single variable, but do not try to find the maximum value.