Mathematics 2443-004 | Name (please print) ______________________
Examination II Form A | Student Number ______________
March 10, 2000

I. (11) Let D be the closed unit disc consisting of all (x, y) with $x^2 + y^2 \leq 1$. A lamina occupies the portion of D where $y \leq 0$ (that is, the bottom half of D). Assume that the lamina has density at (x, y) proportional to the distance from the line $x = 2$.

1. Write an expression for the density function $\rho(x, y)$.

2. Write an integral whose value is the mass of the lamina. Supply the limits of integration and the expression for dA needed to calculate the integral, if one integrates first with respect to x and then with respect to y, but *do not* carry out the calculation.

3. Write a quotient of two integrals which equals the y-coordinate of the center of mass of the lamina. Supply the limits of integration and other information needed for the calculation of the integrals using polar coordinates, but *do not* carry out the calculation.

II. (8) Let D be the unit disc consisting of all (x, y) for which $x^2 + y^2 \leq 1$. Let $f(x, y) = x^2 - y^2$. Use the methods of section 12.7 (i. e. critical points and investigation of the values of f to the boundary of D) to determine the maximum value of f on D, and the point or points at which it occurs.

III. (4) Let C be the unit circle where $x^2 + y^2 = 1$. Let $f(x, y) = x^2 - y^2$. Write the (three) equations that would need to be solved if the method of Lagrange multipliers is to be used to find the maximum value of f on C, but do *not* try to solve the equations or proceed further with finding the maximum.

IV.
(10)

1. Draw a (reasonably large-sized) picture showing a portion of the graph $z = f(x, y)$ for a typical function $f(x, y)$, where $f(x, y) > 0$, which has domain R consisting of all (x, y) with $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

2. Using the picture for part (a), draw a typical cross-section of the region under the graph and above the xy-plane for a fixed value y_0 of y. Write an expression for its area.

3. Using the picture for part (a), draw a vector $\vec{v}$ tangent to the graph $z = f(x, y)$ at a typical point $(x_0, y_0, f(x_0, y_0))$, so that the $\vec{\imath}$ component of $\vec{v}$ is $0 \cdot \vec{\imath}$ and the the $\vec{\jmath}$ component of $\vec{v}$ is $1 \cdot \vec{\jmath}$. Write an expression for the $\vec{k}$ component of $\vec{v}$.

V. State the Extreme Value Theorem for functions of two variables.
(3)

VI. Write an integral in spherical coordinates whose value is the volume of the upper hemisphere of radius R
(7) where $x^2 + y^2 + z^2 \leq R^2$, $z \geq 0$. Carry out the integration to determine that the volume is $2\pi R^3/3$.

VII. Evaluate the following integral by reversing the order of integration: $\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy$.
(7)