Mathematics 4433-001

Examination I

February 11, 2000

I. (5) Take as given the fact that the sine function is continuous. Making use of a theorem we proved in class, prove that there exists a number a so that $\sin(a) = 0.776$.

II. (8) Let $f\colon (0,\infty) \to \mathbb{R}$ be defined by $f(x) = \dfrac{1}{x}$.

(a) Prove that $\lim_{x\to\infty} f(x) = 0$.

(b) Prove that $\lim_{x\to 0} f(x) = \infty$.

III. (6) Let $f\colon \mathcal{D}(f) \to \mathbb{R}$. Let x_0 be a point in $\mathcal{D}(f)$, and let A be a subset of $\mathcal{D}(f)$. Give precise definitions of the following.

(a) f is continuous at x_0

(b) f is continuous on A

(c) f is uniformly continuous on A

IV. (10) Define what it means to say that a subset U of $\mathbb{R}$ is *open.* Define what it means to say that a subset X of $\mathbb{R}$ is *bounded.* Define what it means to say that a subset X of $\mathbb{R}$ is *compact.* Prove that if X is compact, then X is bounded.

V. (5) Write the epsilon-delta definition of the statement that f is *not* continuous at x_0.

VI. (6) Sketch the graph of the following function: $f\colon \mathbb{R} \to \mathbb{R}$, $f(x) = x + 1$ if $x \leq 2$, $f(x) = x$ if $2 < x$. Give a specific open subset U of $\mathbb{R}$ whose preimage $f^{-1}(U)$ is not open. Tell what its preimage set is, and verify that it is not open.

VII. (5) Let $f\colon [0,1] \to [0,1]$ be continuous. Making use of a major theorem that we proved in class, prove that there is a number $c \in [0,1]$ such that $f(c) = c$.

VIII. (6) Prove that the function $f\colon (0,1] \to \mathbb{R}$ drawn here is not uniformly continuous.