

I. Solve the initial value problem $y' = xe^{-x}$, $y(0) = 3$.

(4)

II. Give two different solutions to the IVP $y' = y^{2/3}$, $y(0) = 0$. You need not check that they are solutions.

(2)

III. Give two different solutions to the IVP $y' = y^{2/3}$, $y(2) = 1$. You need not check that they are solutions.

(2)

IV. For the IVP $y' = y^{2/3}$, $y(1) = 1$, tell why any two solutions must agree on some interval containing the number $x = 1$.

(2)

V. Write down a function $f(x)$ whose derivative is e^{x^2} .

(3)

VI. Solve $y'' + y' = 0$, $y(0) = -2$, $y'(0) = 3$.
(7)

VII. Find a general solution of $x^2y' = xy + x^2e^{-y/x}$.
(5)

VIII. Use the Bernoulli substitution $v = y^{1-r}$ to find a general solution to $y' + y^3 = y$.
(6)

IX. Calculate the Wronskian of $e^x \cos(x)$ and $e^x \sin(x)$.
(3)

X. Show that the functions x and $|x|$ are linearly independent on the interval $I = (-2, 2)$ as follows: Suppose they are linearly dependent, so that there are constants k_1 and k_2 , not both zero, with $k_1x + k_2|x| = 0$ for all values of x in I . Now, use particular values of x to reach a contradiction.
(3)

XI. Use the method of separation of variables to find all solutions to the differential equation $y' = y^2$. Solve
(6) the initial value problem $y' = y^2$, $y(-1) = -2$.

XII. Tell what it means to say that a second-order linear equation $A(x)y'' + B(x)y' + C(x)y = F(x)$ is *homo-*
(4) *geneous*. State the Principle of Superposition.

XIII. On what interval does the Existence and Uniqueness Theorem for second-order linear equations guarantee
(3) the existence of a solution to $xy'' + y' + \frac{x}{10-x}y = 0$, $y(1) = 2$, $y'(1) = -3$? (Show some explanation of how you arrived at your answer.)

XIV. Bonus: Find a solution to $x^2y'' + xy' + y = x^2 + x + 1$.
(4)