

I. Let C be the circle $x^2 + y^2 = 1$ in the xy -plane, oriented counterclockwise. Regard C as parameterized (12) by the vector-valued function $\vec{r}(t) = \cos(t)\vec{i} + \sin(t)\vec{j}$, $0 \leq t \leq 2\pi$. Note that the velocity vector $\vec{r}'(t) = -\sin(t)\vec{i} + \cos(t)\vec{j} = -y\vec{i} + x\vec{j}$ has length 1, so it equals the unit tangent vector \vec{T} .

1. Calculate $\int_C (y\vec{i} - x\vec{j}) \cdot d\vec{r}$, directly from the definition of the line integral of a vector field.

2. Calculate $\int_C (y\vec{i} - x\vec{j}) \cdot d\vec{r}$, by direct calculation using the fact that $\int_C (P\vec{i} + Q\vec{j}) \cdot d\vec{r} = \int_C P dx + Q dy$.

3. Calculate $\int_C (y\vec{i} - x\vec{j}) \cdot d\vec{r}$, using the interpretation of the line integral of a vector field as an integral involving the expression $\vec{F} \cdot \vec{T}$.

4. Calculate $\int_C (y\vec{i} - x\vec{j}) \cdot d\vec{r}$, using Green's Theorem.

II. Tell what Clairaut's Theorem says.
(3)

III. Let S be the surface given by $x = u \cos(v)$, $y = u \sin(v)$, and $z = u$, where the domain of the parameterization is the rectangle $0 \leq u \leq 1$ and $0 \leq v \leq 2\pi$.
(9)

1. Carry out the steps in the computation that $\|\vec{r}_u \times \vec{r}_v\| = \sqrt{2}u$.

2. Calculate $\iint_S x^2 dS$.

3. Calculate $\iint_S z^2 \vec{k} \cdot d\vec{S}$.

- IV.** State Stokes' Theorem (not all the hypotheses, just the main formula). Use it to calculate the line integral
(8) $\int_C (x\vec{i} + y\vec{j} + (y^2 - x^2)\vec{k}) \cdot d\vec{r}$, where C is the boundary of the part of the paraboloid $z = 1 - x^2 - y^2$ that lies in the first octant (assume that C is oriented counterclockwise when viewed from above).

- V.** Verify that the Divergence Theorem is true for the vector field $\vec{F}(x, y, z) = x\vec{i} + y\vec{j} + z\vec{k}$ on the region E
(8) which is the unit ball $x^2 + y^2 + z^2 \leq 1$.

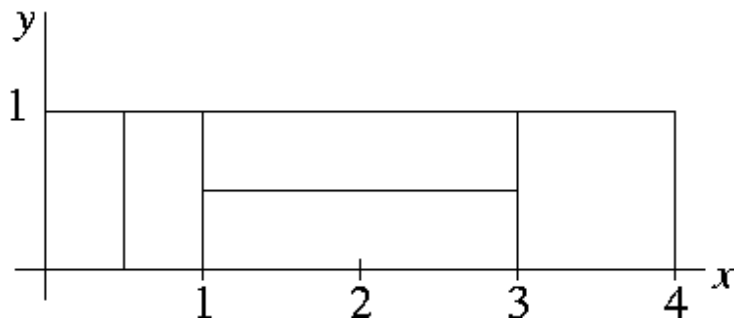
VI. Use the Fundamental Theorem of Calculus to carry out a partial calculation of $\iint_R \frac{\partial P}{\partial x} dA$, where R is the
(3) rectangle $-1 \leq x \leq 1$, $0 \leq y \leq 2$, and $P(x, y)$ is a function of x and y .

VII. Find a vector field \vec{F} in the plane so that if C is any path in the plane, and C starts at P and ends at
(3) Q , then $\int_C \vec{F} \cdot d\vec{r}$ equals the product of the x - and y -coordinates of Q , minus the product of the x - and y -coordinates of P .

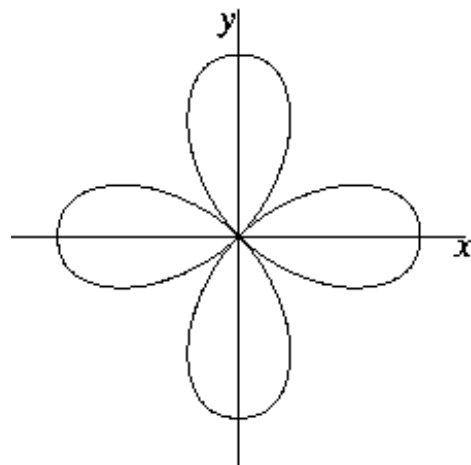
VIII. Use the gradient to find the directional derivative of $f(x, y) = \sin(xy)$ at the point $(2, 1)$ in the direction
(4) toward $(3, -1)$.

IX. Use implicit differentiation to calculate $\frac{\partial R}{\partial R_1}$ if $\frac{1}{R^3} = \frac{1}{R_1^2} + \frac{1}{R_2^2} + \frac{1}{R_3^2}$.
(3)

- X.** Give the Riemann sum that approximates the integral $\int_0^1 \int_0^4 xy \, dx \, dy$ using the partition of the domain shown to the right, and taking the function values at the midpoints of the rectangles. Leave the answer as a sum of fractions, do not compute their sum.



- XI.** The figure to the right shows the graph of the polar equation $r = \cos(2\theta)$. Use a double integral in polar coordinates to calculate the area contained inside each one of its loops.



- XII.** Let E be the portion of the unit ball that has its x and y -coordinates positive, that is, the region $x^2 + y^2 + z^2 \leq 1$, $x \geq 0$, $y \geq 0$. Let S be the boundary surface of E , with the outward normal. Use the Divergence Theorem to calculate $\iint_S (x^3 \vec{i} + y^3 \vec{j} + z^3 \vec{k}) \cdot d\vec{S}$.

XIII. For the function $f(x, y) = \sqrt{x^2 + y^2} - \frac{1}{2} \ln(x^2 + y^2)$, find all critical points of f .
(4)

XIV. A certain function $f(x, y)$ has gradient $e^{x^2} \vec{i} + y \cos(y^2) \vec{j}$. Its value at $(0, 0)$ is 6. Find its value at $(0, 1)$.
(4) (Note: you cannot integrate e^{x^2} , so you cannot calculate an explicit expression for $f(x, y)$).

XV. Let C be the portion of the unit circle that lies in the first quadrant. Find the maximum value of the
(4) function $f(x, y) = x^2 y$ on C by parameterizing C as $x = \cos(\theta)$, $y = \sin(\theta)$ and regarding f as a function of θ .