Mathematics 2443-005 Name (please print) ______________________

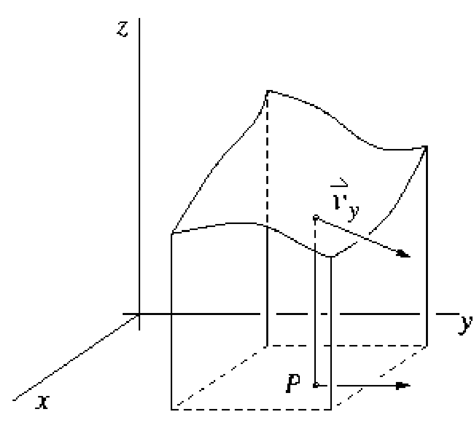
Examination II Form A

March 6, 2003

I. (5) For the following integral, sketch the region of integration and change the order of integration. The answer should have two terms. $\displaystyle\int_0^1 \int_{2x}^{4x} f(x,y)\,dy\,dx$

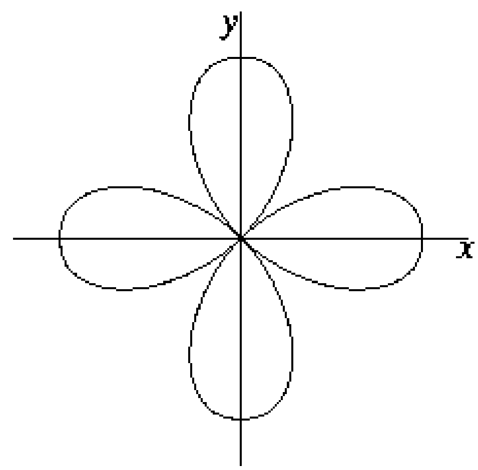
II. (4) For the rectangle $R = [0,1] \times [0,2]$, calculate $\iint_R \dfrac{xy}{\sqrt{1+x^2+y^2}}\,dA$.

III. (3) The figure to the right shows the portion of the graph of a certain function $f(x,y)$, and a certain point P in the domain of f. Also shown are the vector $\vec{\jmath}$, located at P, and a vector $\vec{v}_y$ tangent to the surface at the point directly above P. Suppose that f_x has the value -0.67 at P and f_y has the value -0.65. Find a, b, and c so that $\vec{v}_y = a\vec{\imath} + b\vec{\jmath} + c\vec{k}$.



IV. Calculate $\int_0^1 \int_0^z \int_0^y ze^{-y^2}\, dx\, dy\, dz$.
(4)

V. The figure to the right shows the graph of the polar equation $r = \cos(2\theta)$. Use a double integral in polar coordinates to calculate the area contained inside each one of its loops. You might need to use the identity $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$.
(5)



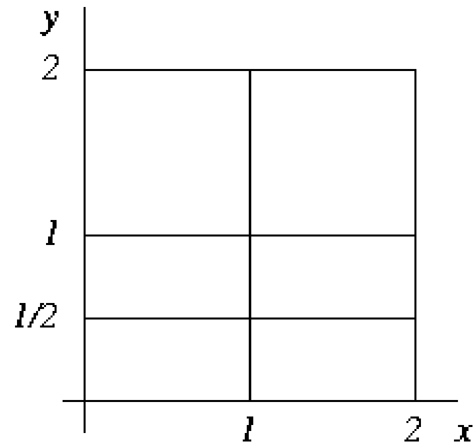
VI. Sketch the region in the first octant bounded by the three coordinate planes and the plane $x + y + z = 1$. Write a triple integral whose value is the volume of this region. Supply limits of integration, but *do not* carry out the calculation to evaluate the integral.
(4)

VII. (5) Find the mass of the upper hemisphere E given by $x^2 + y^2 + z^2 = a^2$, $z \geq 0$ if the density function is z. In spherical coordinates, $x = \rho\cos(\theta)\sin(\phi)$, $y = \rho\sin(\theta)\sin(\phi)$, $z = \rho\cos(\phi)$, and $dV = \rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta$.

VIII. (4) State the Fundamental Theorem of Calculus (without hypotheses, just the formula). Calculate $\dfrac{\partial}{\partial y}\displaystyle\int_0^{x^2y^2} \sin^{100}(t^2)\,dt$.

IX. (4) Evaluate $\iiint_E \sqrt{x^2+y^2}\,dV$, where E is the region that lies inside the cylinder $x^2 + y^2 = 4$ and between the planes $z = -1$ and $z = 2$. Use cylindrical coordinates, so that $\sqrt{x^2+y^2} = r$.

X. (4) Use a Riemann sum for this partition of the rectangle $R = [0,2] \times [0,2]$ to estimate $\iint_R \sqrt{x^2+y^2}\, dA$, choosing as the sample points the points closest to the origin. Leave the Riemann sum as an unsimplified sum of terms, possibly involving square roots.



XI. (5) Consider a lamina that occupies the region of the unit disk in the xy-plane. Suppose that the density at each point is proportional to the square of the distance from the point to the origin. Write an expression for the density function ρ in polar coordinates, and use it to find the mass of the lamina.

XII. (4) Consider the paraboloid $z = x^2 + y^2$ and the saddle surface $z = x^2 - y^2$. Tell how one can know that if D is any domain in the xy-plane, then the areas of the portions of these two surfaces having their (x, y) coordinates in D are equal.