Mathematics 2443-006H Name (please print) ____________________

Examination III

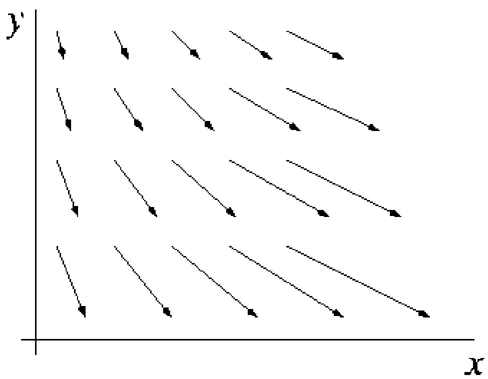
April 27, 2006

Instructions: Give brief, clear answers. Use Green's Theorem whenever possible.

I. (6) Let $\vec{F}$ be the vector field $\dfrac{-y}{x^2+y^2}\vec{\imath} + \dfrac{x}{x^2+y^2}\vec{\jmath}$. Verify by calculation (direct or indirect) that $\int_C \vec{F}\cdot d\vec{r}$ is not path-independent on the domain $\{(x,y) \mid (x,y) \neq (0,0)\}$.

II. (12) (a) Evaluate the line integral $\int_C xy\,dx + x^2y\,dy$ directly, where C is the triangle with vertices $(0,0)$, $(1,0)$, and $(1,2)$.
(b) Evaluate it using Green's Theorem.

III. (7) The figure to the right shows a vector field $P\vec{\imath} + Q\vec{\jmath}$ on a portion of the plane. Based on its appearance there:



(a) Explain geometrically why $\dfrac{\partial P}{\partial x}$ is positive.
(b) Explain geometrically why $\dfrac{\partial P}{\partial y}$ is negative.
(c) Explain geometrically why $\dfrac{\partial Q}{\partial x}$ is zero.
(d) Explain geometrically why $\dfrac{\partial Q}{\partial y}$ is positive.
(e) Determine whether $\operatorname{div}(P\vec{\imath} + Q\vec{\jmath})$ is positive or negative.
(f) Determine whether $\operatorname{curl}(P\vec{\imath} + Q\vec{\jmath})\cdot\vec{k}$ is positive or negative.

IV. (6) Let f be a scalar function of three variables and let $\vec{F}$ be a vector field on a 3-dimensional domain. Writing $\vec{F}$ as $P\vec{\imath} + Q\vec{\jmath} + R\vec{k}$, verify that $\operatorname{div}(f\vec{F}) = f\operatorname{div}(\vec{F}) + \vec{F}\cdot\nabla f$.

V. (6) (a) Sketch the vector field $\dfrac{-y}{x^2+y^2}\,\vec{\imath} + \dfrac{x}{x^2+y^2}\,\vec{\jmath}$.

(b) Explain the important phenomenon (related to Clairaut's Theorem) that the vector field $\dfrac{-y}{x^2+y^2}\,\vec{\imath} + \dfrac{x}{x^2+y^2}\,\vec{\jmath}$ illustrates.

VI. (5) Let D be a connected planar domain.
(a) Define what it means to say that D is simply-connected (do better than "no holes").
(b) State the theorem discussed in class that uses the hypothesis that D is simply-connected.

VII. (6) Suppose that $\vec{F} = P\vec{\imath} + Q\vec{\jmath}$ is a vector field on the plane and let C be the unit circle. Suppose that at points on C (but not necessarily on the rest of D), $\vec{F}(x,y) = x\vec{\imath} + y\vec{\jmath}$.
(a) Verifty that on C, $\vec{F}$ equals the outward unit normal $\vec{n}$.
(b) Calculate $\displaystyle\iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\,dA$.
(c) Calculate $\displaystyle\iint_D \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}\,dA$.

VIII. (6) Calculate $\int_C (y\vec{\imath} - x\vec{\jmath})\cdot d\vec{r}$, where C is the equilateral triangle that has one side equal to the straight line from $(1,1)$ to $(201,1)$ but does not lie completely in the first quadrant.