Math 2513 homework

28. (4/13) 2.4 # 28, 30, 38-40

29. (4/13) Prove the following.
(a) $\forall a \in \mathbb{Z}, a \equiv a \mod m$.
(b) $\forall a \in \mathbb{Z}, \forall b \in \mathbb{Z}, a \equiv b \mod m \Rightarrow b \equiv a \mod m$.
(c) $\forall a \in \mathbb{Z}, \forall b \in \mathbb{Z}, \forall c \in \mathbb{Z}, (a \equiv b \mod m \wedge b \equiv c \mod m) \Rightarrow a \equiv c \mod m$.

30. (4/13) Let p be prime.
(a) Prove that $\gcd(a, p) = 1$ if and only if p does not divide a.
(b) Using a fact presented in class, prove that if $c \not\equiv 0 \mod p$ and $ac \equiv bc \mod p$, then $a \equiv b \mod p$. (That is, when working "mod a prime", one *can* always cancel nonzero common factors from both sides of an equation.)

31. (4/13) 2.5 # 22

32. (4/13) 3.2 # 31

33. (4/13) 3.2 # 38 (arrange the pairs (m, n) analogously to the way we arranged the positive fractions when proving that $\mathbb{Q}$ is countable)

34. (4/13) Let S be the set of sequences of 0's and 1's, $S = \{a_1 a_2 a_3 \cdots \mid a_i \in \{0, 1\}\}$. A typical element of S is $00101101100011010\cdots$. Adapt the proof that $\mathbb{R}$ is uncountable to prove that S is uncountable.

35. Let X be a set.
(a) Prove that there does not exist a surjective function from X to $\mathcal{P}(X)$.
(b) Prove that if X is nonempty, then there exists a surjective function from $\mathcal{P}(X)$ to X.

36. 3.3 # 7, 10, 12, 20 (give 3 proofs: 1. using induction, 2. using congruence, and 3. deduce it from the fact, proven in class, that $3 | n^3 - n$), 23, 41, 52

37. 4.1 # 2, 3, 7-9

38. 4.1 # 28, 29, 32, 33, 36

39. 4.2 # 3, 8

40. 4.2 # 13, 14, 20, 21, 22, 31

41. 4.3 # 1, 8, 9, 25, 26

42. 4.4 # (don't worry about these for the final exam, do them for fun over the summer) 3, 4, 12, 21a), 30, 31 Hint: # 31 follows almost immediately from the identity $0 = \sum_{k=0}^{n} (-1)^k \binom{n}{k}$