Mathematics 2513-001 Name (please print) ______________________

Examination II Form A

March 23, 2006

Instructions: Give brief, clear answers. "Prove" means "give an argument". In giving definitions, give the *precise* definition, using logical notation and/or set notation as appropriate.

I. (2) Write the following as an implication: "$a^2 \geq 2$ for at most one a".

II. (4) Let $T(p, c)$ be "Person p has traveled to the city c." Write each of the following statements in logical notation, putting in all necessary quantifiers using the sets $\mathcal{P}$ of all people and $\mathcal{C}$ of all destination cities. If your answer involves a negation, simplify as much as possible.
(a) Jeff has been to Madrid or Paris.
(b) No one has traveled to every city.
(c) Everyone has traveled to at least one city.
(d) Any two people have traveled to at least one city in common.

III. (4) Write out all elements of $\mathcal{P}(\{a, b\})$ and all elements of $\mathcal{P}(\{a, b\}) \times \{a, b, c\}$.

IV. (10) For the function $G\colon s \to t$ (where s and t are sets), give definitions of the following. As requested in the instructions, give the *precise* definitions, using logical notation and/or set notation as appropriate.
(a) the range of G
(b) the preimage of an element T of t
(c) the inverse function G^{-1}, assuming that G is bijective
(d) the composition $H \circ G$, assuming that $H\colon t \to u$ (part of giving the definition of a function is telling its domain and codomain).
(e) $G = K$, where $K\colon u \to v$
(f) the graph of G

V. (4) Let X be an infinite set.
(a) Define what it means to say that X is *countable.*
(b) Show a function that verifies that $\mathbb{Z}$ is countable.

VI. (4) Let $f\colon X \to Y$ and $g\colon Y \to Z$. Prove that if f and g are injective, then the composition $g \circ f$ is injective.

VII. (8) Let $f\colon (0, \pi) \to (0, \infty)$ be the function defined by $f(x) = \csc(x)$, where the cosecant function is as usual given by $\csc(x) = \dfrac{1}{\sin(x)}$.
(a) Prove that f is not injective.
(b) Prove that f is not surjective.

VIII. (4) Let $f\colon \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ be the function defined by $f(m, n) = m^2 - n$. Prove that f is surjective.

IX. (3) Disprove the following assertion: for all sets A, B, and C, if $A \cap B = A \cap C$, then $B = C$.

X. (4) Give an example of a function from $\mathbb{R}$ to $\mathbb{R}$ that is injective but not surjective.

XI. (3) Let $A = \mathbb{R}$ and $B = \mathbb{Z}$. Give examples of each of the following.
(a) An element of $A \times B$ that is not in $B \times B$.
(b) An element of $B \times A$ that is not in $B \times B$.
(c) An element of $A \times A$ that is neither in $A \times B$ nor in $B \times A$.