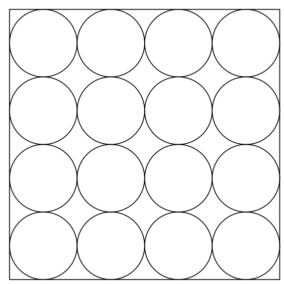
Math 2423 homework

1. (due 2/1) Give a simple formula for $\sum_{k=0}^{n}(-1)^k x^k$ (the answer involves the expression $(-1)^n$).

2. (2/1) Suppose you take a square of side s and inscribe n^2 congruent circles, as shown in this figure for $n = 4$:



Try to decide, intuitively, whether the total area inside the circles converges to the area inside the square, as $n \to \infty$. Calculate the area inside the circles, as a function of n, and take the limit to see whether your intuition was correct.

3. (2/1) Know the following from memory: the Intermediate Value Theorem, the Extreme Value Theorem, the Mean Value Theorem.

4. (2/1) Review the Chain Rule, and work enough problems to be sure that you can use it perfectly.

5. (2/1) Let f be a function which is differentiable everywhere. For the error term $E(h)$ in $f(a+h) = f(a)+f'(a)h+E(h)$, use the Mean Value Theorem to obtain the estimate that for some c between a and $a+h$, $|E(h)| \le |f''(c)|h^2$.

6. (2/1) Use the previous problem to show that $|\sin(x) - x| \le x^2$ for all x.

7. (2/1) Use the telescoping sum $\sum_{k=1}^{n} k^4 - (k-1)^4$ and the formulas that we established for $\sum_{k=1}^{n} k$ and $\sum_{k=1}^{n} k^2$ to obtain the formula $\sum_{k=1}^{n} k^3 = \dfrac{n^2(n+1)^2}{4}$.

8. (2/1) 5.1 # 20-22