Mathematics 2423-001H | Name (please print) ______________________

Examination I

February 13, 2007

Instructions: Give brief, clear answers. It is not expected that most people will be able to answer all the questions, just do what you can in 75 minutes. All integrals may be assumed to exist— there are no tricks involving functions that are not continuous on the domain of integration.

I. (24) Determine the values of the following integrals:

1. $\displaystyle\int \sin(\pi t)\, dt$
2. $\displaystyle\int_{\pi/4}^{\pi/2} \csc^2(w)\, dw$
3. $\displaystyle\int \sec^5(\theta)\tan(\theta)\, d\theta$
4. $\displaystyle\int_0^3 t\sqrt{9-t^2}\, dt$
5. $\displaystyle\int_0^3 \sqrt{9-t^2}\, dt$
6. $\displaystyle\int \frac{x}{\sqrt{x+2}}\, dx$

II. (6) Calculate the following derivatives:

1. $\displaystyle\frac{d}{dx}\int_x^1 \sqrt{\tan(t)}\, dt$
2. $\displaystyle\frac{d}{dx}\int_{\tan(x)}^1 \sqrt{\tan(t)}\, dt$

III. (5) Explain (briefly and concisely) the terms *partition* and *Riemann sum,* and how they are used to define the definite integral $\displaystyle\int_a^b f(x)\, dx$.

IV. (3) Explain geometrically why the integral of the rate of change of a function should equal the total net change of a function, that is, why $\displaystyle\int_a^b f'(x)\, dx = f(b) - f(a)$. (This is, of course, the second assertion of the Fundamental Theorem of Calculus.)

V. (6) As you know, the natural logarithm function is defined to be $\ln(x) = \displaystyle\int_1^x \frac{1}{t}\, dt$. In particular, if a and b are any numbers larger than 1, then $\ln(ab) = \displaystyle\int_1^{ab} \frac{1}{t}\, dt$.

(a) Use the substitution $u = t/a$ to show that $\displaystyle\int_a^{ab} \frac{1}{t}\, dt = \int_1^b \frac{1}{u}\, du = \ln(b)$.

(b) Use part (a) and additivity of the integral on domains to verify that $\ln(ab) = \ln(a) + \ln(b)$.

VI. (3) Find the value c that satisfies the Mean Value Theorem for Integrals for the function $f(x) = x^3$ on the interval $[0, 2]$.

VII. (4) Let $g(x)$ be the function on the interval $[0, 10]$ defined by $g(x) = 0$ if x is rational, and $g(x) = 1$ if x is irrational. Partition $[0, 10]$ into two equal subintervals.

1. Calculate Δx.
2. Give three explicit choices of sample points x_1^* and x_2^* so that the Riemann sum $\displaystyle\sum_{k=1}^{2} g(x_i^*)\,\Delta x$ has different values for the three choices.

VIII. (4) Use the telescoping sum $x^{n+1} - 1 = \displaystyle\sum_{k=1}^{n+1} x^k - x^{k-1}$ to verify that $(x^n + x^{n-1} + \cdots + x + 1)(x - 1) = x^{n+1} - 1$.