Instructions: Give brief, clear answers. It is not expected that most people will be able to answer all the questions, just do what you can in 75 minutes.

I. Let R be the region bounded by $y = e^{-x^2}$, $x = 0$, and $x = 2$.
(7)

1. Calculate the volume produced when this region is rotated about the y-axis.

2. Write an integral whose value is the volume produced when R is rotated about the line $y = 2$, but do *not* evaluate it.

II. Calculate the following derivatives:
(6)

1. $\dfrac{d}{dx}(5^{-1/x})$

2. $\dfrac{d}{dx}(\log_3(x^2 - 4))$

III. Use the definition of $\ln(x)$ and the fact that integration is additive on unions of domains to verify that
(5) $\ln(ab) = \ln(a) + \ln(b)$.

IV. The following problem concerns the function $\sin^{-1}(x)$, which is the inverse function of the function $f(x)$
(5) with domain $[-\frac{\pi}{2}, \frac{\pi}{2}]$ given by $f(x) = \sin(x)$.

1. Draw a right triangle containing an angle of $\sin^{-1}(x)$, and use it to find $\cos(\sin^{-1}(x))$.

2. Differentiate the equation $\sin(\sin^{-1}(x)) = x$ and simplify to find the derivative of $\sin^{-1}(x)$.

V. Give a precise definition of what it means to say that a function f is *injective* (also called *one-to-one*).
(3)

VI. Calculate the area of the region bounded by $y = \sin^{-1}(x)$, $x = 1$, and $y = 0$.
(4)

VII. Find the domain of the function $\ln(e^x - 2)$.
(3)

VIII. Evaluate the following integrals.
(12)

1. $\displaystyle\int \frac{e^x + 1}{e^x}\, dx$

2. $\displaystyle\int \frac{e^x}{e^x + 1}\, dx$

3. $\displaystyle\int \frac{t^2}{5 + t^6}\, dt$

4. $\displaystyle\int \frac{\sin^{-1}(x)}{\sqrt{1 - x^2}}\, dx$

IX. Solve for x in the equation $2^{ax} = \ln(c)\, 3^{bx}$.
(4)

X.
(6) A painting in an art gallery has height h and is hung so that its lower edge is a distance d above the eye of an observer. How far from the wall should the observer stand so as to get the best view (that is, so that the angle θ is largest)?