Mathematics 2423-001H Name (please print) ______________

Examination III

April 26, 2007

I. (20) Evaluate the following integrals:

1. $\int \sin^3(3x)\,\cos^2(3x)\,dx$

2. $\int \sin^2(x)\,\cos^2(x)\,dx$

3. $\int \sin^{-1}(x)\,dx.$

4. $\int \frac{x^3}{x^2+1}\,dx.$

5. $\int \sinh(\sqrt{x})\,dx.$

II. (6) Calculate the following derivatives:

1. $\frac{d}{dx}(5^{-1/x})$

2. $\frac{d}{dx}(\log_3(x^2-4))$

III. (6) Use the method of partial fractions to evaluate $\int \frac{1}{x^4+x^2}\,dx.$

IV. (6) Use the substitution $x = \tan(\theta)$ to evaluate $\int \frac{1}{x^4+x^2}\,dx$. Make it clear how you are obtaining the answer in terms of x.

V. (4) On a single coordinate system, sketch the graphs of $y = \sinh(x)$, $y = \cosh(x)$, and $y = e^x/2$. Make the relation between them clear.

VI. (6) Evaluate the following limits. Show your reasoning.

1. $\lim_{x\to\infty} x^{\ln(2)/(1+\ln(x))}$

2. $\lim_{h\to 0} \frac{f(x+h)-2f(x)+f(x-h)}{h^2}$

VII. (8) Suppose that $f(x)$ is a function whose second derivative $f''(x)$ exists and is continuous. Define $E(h)$ by the formula $f(a+h) = f(a) + f'(a)h + E(h)$. That is, $E(h)$ is the error of linear approximation.

1. Use integration by parts to calculate that $E(h) = \int_0^h (h-t)\,f''(a+t)\,dt$.

2. Let m be the minimum and M the maximum of f'' on the interval $[a, a+h]$, so that

$$m \le f''(a+t) \le M$$

for $0 \le t \le h$. Notice that

$$(h-t)\,m \le (h-t)\,f''(a+t) \le (h-t)\,M$$

for $0 \le t \le h$. Now, show that $\frac{1}{2}h^2\,m \le E(h) \le \frac{1}{2}h^2\,M$.

3. Use the Intermediate Value Theorem to show that there exists c in $[a, a+h]$ so that $E(h) = \frac{1}{2}f''(c)\,h$.