

Mathematics 6843 – Surfaces, Mapping Class Groups, and Curve Complexes

Course Outline

- I. The topology of surfaces
 1. Examples
 2. Orientation
 3. Classification up to homeomorphism
- II. Curves in surfaces
 1. Classification up to homeomorphism
 2. Algebraic and geometric intersection
 3. The case of the torus
 4. The Farey graph
- III. The curve complex
 1. Definition and basic facts
 2. The curve complex is connected
- IV. Mapping class groups
 1. Dehn twists
 2. The case of the torus
 - i. $\mathcal{M}(T) \rightarrow \mathrm{SL}(2, \mathbb{Z})$
 - ii. Anosov homeomorphisms
 - iii. Action on the Farey graph
 - iv. Classification of mapping classes
 3. Dehn twists generate
 - i. The planar case - Lickorish's classical method
 - ii. The general case - Proof using the curve complex
- V. Hyperbolic structures on surfaces
 1. Hyperbolic space and its isometries
 2. Hyperbolic structures on the annulus, cusps
 3. Hyperbolic structures on pair of pants
 4. The general case
 5. Geodesic loops in hyperbolic surfaces
- VI. Teichmüller space
 1. Markings and Riemannian metrics
 2. The Fenchel-Nielsen flow
 3. Kerckhoff's Cosine Formula
- VII. Measured foliations
 1. Examples
 2. Singular Euclidean structures
 3. Weighted curve systems and rational foliations

4. Train tracks and projective measured foliations
5. Geodesic laminations

VIII. The Nielsen-Thurston Classification Theorem

1. PseudoAnosov homeomorphisms
2. The Thurston boundary
3. The proof of the Classification Theorem

IX. The curve complex returns

1. Deformations of hyperbolic structures
2. The curve complex meets Teichmüller space
3. The curve complex is Gromov negatively curved
4. The curve complex has infinite diameter

Information on the curve complex can be found in:

1. S. Schleimer, *Notes on the Complex of Curves*, available at <http://www.warwick.ac.uk/~masgar/math.html> .
2. N. Ivanov, Mapping class groups, in *Handbook of Geometric Topology*, 523–633, North-Holland, Amsterdam, 2002.
3. Y. Minsky, Curve complexes, surfaces and 3-manifolds, *International Congress of Mathematicians, Vol. II*, 1001–1033, Eur. Math. Soc., Zürich, 2006.
4. Many research articles, notably:
 - (a) H. Masur and Y. Minsky, Geometry of the complex of curves. I. Hyperbolicity, *Invent. Math.* 138 (1999), no. 1, 103–149.
 - (b) H. Masur and Y. Minsky, Geometry of the complex of curves. II. Hierarchical structure, *Geom. Funct. Anal.* 10 (2000), no. 4, 902–974.