Math 6843 homework

1. (due 1/22) Let M be the Möbius band.

   (a) Construct an explicit (in coordinates) deformation retraction from M onto its center circle.

   (b) Prove that M does not retract to ∂M.

2. (1/22) Let M be the Möbius band. Regard its boundary as S^1. Using our definitions, determine the quotient space $M/\langle x \sim y \text{ for all } x, y \in \partial M\rangle$. Give at least a couple of different explanations, for example, one using Euler characteristic and one describing a homeomorphism to one of the manifolds that we discussed in class.

3. (1/22) Let M be the Möbius band. Regard its boundary as S^1. Using our definitions, determine the quotient space $M/\langle x \sim -x \text{ for all } x \in \partial M\rangle$. Give at least a couple of different explanations, for example, one using Euler characteristic and one describing a homeomorphism to one of the manifolds that we discussed in class.

4. (1/24) (a) Regard the punctured Möbius band F as two disks connected by three bands, one of them twisted. Find a loop in the interior of F that bounds a crosscap. (b) Regard the punctured Klein bottle F as two disks connected by three bands, two of them twisted. Find two loops in the interior of F that bound disjoint crosscaps.

5. (1/24) Draw a punctured torus imbedded in $\mathbb{R}^3$ so that its boundary circle is a figure-8 knot.

6. (1/29) Use the Classification Theorem to give an informal (i. e. mostly pictures) argument that every compact, connected orientable surface has an orientation-reversing self-homeomorphism. In fact, show the following:

   1. If F has an even number of boundary circles, then it has an orientation-reversing involution (a homeomorphism h for which $h \circ h$ is the identity) whose fixed-point set is a circle.
   2. If F has an odd number of boundary circles, then it has an orientation-reversing involution whose fixed-point set is an arc.
   3. It F has an even number of boundary circles, then it has an orientation-reversing involution whose fixed-point set is empty. [Rk: If an orientable surface F has an odd number of boundary circles, then every involution F has a fixed point. For if F had a free involution h, the quotient map $F \to F/h$ would be a 2-covering map and hence $\chi(F/h) = \frac{1}{2}\chi(F)$, but F has odd Euler characteristic so $\chi(F/h)$ would not be an integer.]