Mathematics 3333-001 Name (please print) ______________________

Examination II

March 24, 2009

Instructions: Give concise answers, but clearly indicate your reasoning.

I. (6) Use the row operation method to calculate the inverse of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$.

II. (7) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

(a) Show that A and B are row equivalent. Give a list of elementary matrices $E_1, \ldots, E_k$ for which $E_k \cdots E_1 A = B$.

(b) Explain why A and B cannot be column equivalent.

III. (4) Let V be a vector space and let $S = \{v_1, \ldots, v_k\}$ be a subset of V. Recall that span(S) is the set of all linear combinations of element of S, that is, $\{\sum_{i=1}^{k} \lambda_i v_i \mid \lambda_i \in \mathbb{R}\}$. Verify that span$(S)$ is a subspace of V.

IV. (3) Let $W = \{at^2 + bt + c \mid c \geq 0\}$, that is, the set of all polynomials of degree at most 2 and having non-negative constant term. By giving a specific counterexample, show that W is not a subspace of P_2 (the vector space of all polynomials of degree at most 2).

V. (4) Let A be an $m \times n$ matrix and consider the homogeneous system of linear equations given by $AX = 0$. Its solutions form a subset of $\mathbb{R}^n$. Verify that the set of solutions is a subspace of $\mathbb{R}^n$.

VI. (9) Let V be a vector space and let $S = \{v_1, \ldots, v_k\}$ be a subset of V.

(a) Define what it means to say that S is *linearly independent.*

(b) Define what it means to say that S is a *basis* of V.

(c) If V has dimension 6 and S is a subset consisting of five elements of V, what can you say about S, beyond just the fact that it is not a basis?

(d) If V has dimension 6 and S is a subset consisting of seven elements of V, what can you say about S, beyond just the fact that it is not a basis?

VII. (6) Let $V = \mathbb{R}_3$ (the vector space of 1×3 vectors), and let $S = \left\{ \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}, \begin{bmatrix} 2 & 2 & 3 \end{bmatrix}, \begin{bmatrix} 5 & 2 & 3 \end{bmatrix} \right\}$. Test S for linear independence. If it is not linearly independent, write one of its elements as a linear combination of the others.

VIII. (4) If an $n \times n$ nonsingular matrix A is equivalent to a matrix B, then B must also be nonsingular. Why?

IX. (4) If P is a nonsingular $n \times n$ matrix, then its transpose P^T must also be nonsingular. Why?

X. Let V be the vector space of all differentiable functions from the real numbers to the real numbers, with
(6) the usual addition and scalar multiplication operations.

(a) Verify that the subset $\{1, x, x^2, x^3\}$ is a linearly independent subset of V (hint: suppose you have the zero function 0 written as a linear combination of these functions, then take derivatives three times).

(b) The same kind of argument as in (a) can be used to show that the set $\{1, x, x^2, x^3, x^4\}$ is a linearly independent subset of V, and even that the sets $\{1, x, x^2, x^3, \ldots, x^n\}$ are linearly independent for any choice of n (do not try to check these facts). What does this tell us about the dimension of V. Why?