Mathematics 3113-005 Name (please print) ____________________

Quiz 2 Form A

February 11, 2011

Instructions: Give concise answers, but clearly indicate your reasoning.

I. (3) Check whether the initial value problem $\dfrac{dy}{dx} = y^{1/3}$, $y(3) = 0$ satisfies the hypotheses of the Existence and Uniqueness Theorem. What does the theorem tell you about the solutions of this IVP?

$\dfrac{\partial}{\partial y}\left(y^{1/3}\right) = \dfrac{1}{3y^{2/3}}$. Since this is undefined at $(3, 0)$, the Existence and Uniqueness Theorem does not apply to this IVP. It tells us nothing about the solutions of this IVP.

II. (3) For the first-order linear homogeneous DE $y' + P(x)y = 0$, verify that if y_1 and y_2 are solutions, then so is $Ay_1 + By_2$ for any constants A and B.

Since y_1 and y_2 are solutions, $y_1' + P(x)y_1 = 0$ and $y_2' + P(x)y_2 = 0$. Testing $Ay_1 + By_2$, we find

$$\begin{aligned}(Ay_1 + By_2)' + P(x)(Ay_1 + By_2) &= Ay_1' + By_2' + P(x)Ay_1 + P(x)By_2 \\ &= A\,(y_1' + P(x)y_1) + B\,(y_2' + P(x)y_2) = 0\ .\end{aligned}$$

III. (5) For the linear DE $xy' = 2y + x^3\cos(x)$, find an integrating factor, then carry out the recipe to find the general solution. (Hint: If you find yourself needing integration by parts, you have made a computational error along the way. Don't burn time on the calculation until you have the correct integrating factor and have done the algebra correctly.)

Putting the equation into the standard form, we obtain $y' - \dfrac{2}{x}y = x^2\cos(x)$. We find the integrating factor to be

$$\rho(x) = e^{\int -\frac{2}{x}\,dx} = e^{-2\ln(x)} = e^{\ln(x^{-2})} = x^{-2}\ .$$

Multiplying through by $\rho(x)$ and integrating, we obtain

$$\begin{aligned}\frac{1}{x^2}y' - \frac{2}{x^3}y &= \cos(x) \\ \frac{1}{x^2}y &= \sin(x) + C \\ y &= x^2\sin(x) + Cx^2\end{aligned}$$

Checking, we have $y' = 2x\sin(x) + x^2\cos(x) + 2Cx$, and $2y + x^3\cos(x) = 2x^2\sin(x) + 2Cx^2 + x^3\cos(x) = xy'$.

IV. (4) Rewrite the DE $(4x - y)\,y' = y$ as a homogeneous DE, and carry out the substitution $v = \dfrac{y}{x}$ to transform the equation into a DE of the form $v' = F(v, x)$. Simplify and tell what method you would use to solve this DE, but *do not* carry out the method or proceed beyond this point.

We have

$$\begin{aligned}(4x - y)\,y' &= y \\ y' = \frac{y}{4x - y} &= \frac{\frac{y}{x}}{4 - \frac{y}{x}}\end{aligned}$$

From $v = \dfrac{y}{x}$, we have $vx = y$, so $v + x\dfrac{dv}{dx} = \dfrac{dy}{dx}$. Substituting into the homogeneous form, we obtain

$$\begin{aligned}v + x\frac{dv}{dx} &= \frac{v}{4 - v} \\ \frac{dv}{dx} = \frac{1}{x}\left(\frac{v}{4 - v} - v\right) &= \frac{1}{x}\left(\frac{v^2 - 3v}{4 - v}\right)\ .\end{aligned}$$

It is a separable equation, so one would use the method of separation of variables.