Mathematics 3113-005 Name (please print) ______________________

Quiz 3 Form A

February 25, 2011

Instructions: Give concise answers, but clearly indicate your reasoning.

I. (6) Two linearly independent solutions of the DE $y'' + 3y' + 2y = 0$ are e^{-x} and e^{-2x} (do not check these).

(a) Write a general solution of $y'' + 3y' + 2y = 0$.

$y = Ae^{-x} + Be^{-2x}$.

(b) Find the solution that satisfies $y(1) = 1$, $y'(1) = 0$.

$y' = -Ae^{-x} - 2Be^{-2x}$. We want

$$1 = y(1) = Ae^{-1} + Be^{-2}$$
$$0 = y'(1) = -Ae^{-1} - 2Be^{-2}$$

Adding the two equations gives $1 = -Be^{-2}$, so $B = -e^2$, and from the first equation $1 = Ae^{-1} - e^2e^{-2} = A/e - 1$, so $A = 2e$. The desired solution is $2e\, e^{-x} - e^2\, e^{-2x} = 2e^{1-x} - e^{2-2x}$.

II. (2) This problem concerns the DE $y'' + y - x = 0$. The function $\sin(x) + x$ is a solution, but $2(\sin(x) + x)$ is not. Why does this not violate the Principle of Superposition?

The DE is not homogeneous (put into the standard form, it becomes $y'' + y = x$).

III. (3) This problem concerns the DE $y'' - 2y' + 2 = 2x$.

(a) Write the associated homogeneous equation of $y'' - 2y' + 2 = 2x$.

$y'' - 2y' = 0$.

(b) A solution of $y'' - 2y' + 2 = 2x$ is $x + 1$ (do not check this). Given that $e^x \cos(x)$ and $e^x \sin(x)$ are linearly independent solutions of the associated homogeneous equation, write a general solution of $y'' - 2y' + 2 = 2x$.

$Ae^x \cos(x) + Be^x \sin(x) + x + 1$.

IV. (4) For the DE $4y'' + 4y' + y = 0$, the characteristic equation is $4r^2 + 4r + 1 = (2r + 1)^2$. Since it has repeated roots $-1/2$ and $-1/2$, two solutions of the DE are $e^{-x/2}$ and $xe^{-x/2}$ (do not check that they are solutions). Compute the Wronskian of $e^{-x/2}$ and $xe^{-x/2}$.

$(e^{-x/2})' = -e^{-x/2}/2$ and $(xe^{-x/2})' = e^{-x/2} - xe^{-x/2}/2$, so

$$W(e^{-x/2}, xe^{-x/2}) = \det \begin{pmatrix} e^{-x/2} & xe^{-x/2} \\ -e^{-x/2}/2 & e^{-x/2} - xe^{-x/2}/2 \end{pmatrix} = e^{-x} - xe^{-x}/2 - (-xe^{-x}/2) = e^{-x}\ .$$