Instructions: Give concise answers, but clearly indicate your reasoning.

I.   Define what it means to say that a collection of functions $\{y_1, y_2, \ldots, y_n\}$ is *linearly independent*.
(3)

II.   Show that the set of functions $\{1, \sin^2(x), 2\cos^2(x)\}$ is linearly dependent.
(3)

III.   Given that
(4)
$$\lambda^6 + 2\lambda^4 + 20\lambda^3 + \lambda^2 + 20\lambda + 100 = (\lambda + 2)^2(\lambda^2 - 2\lambda + 5)^2 \ ,$$

write a general solution of the DE

$$y^{(6)} + 2y^{(4)} + 20y^{(3)} + y'' + 20y' + 100y = 0 \ .$$

IV.   The function $\sin(x)$ satisfies the DE $y'' + y' + y = \cos(x)$. Find a general solution.
(5)