Mathematics 3113-005 Name (please print) ______________________

Quiz 4 Form B

March 4, 2011

Instructions: Give concise answers, but clearly indicate your reasoning.

I. Given that
(4)

$$\lambda^6 + 14\lambda^5 + 83\lambda^4 + 268\lambda^3 + 499\lambda^2 + 510\lambda + 225 = (\lambda + 3)^2(\lambda^2 + 4\lambda + 5)^2 \ ,$$

write a general solution of the DE

$$y^{(6)} + 14y^{(5)} + 83y^{(4)} + 268y^{(3)} + 499y'' + 510y' + 225y = 0 \ .$$

The roots of the characteristic polynomial are -3, -3, $-2 \pm i$, and $-2 \pm i$. The first two give the solutions e^{-3x} and xe^{-3x}, and the second two pairs give the solutions $e^{-2x}\cos(x)$, $e^{-2x}\sin(x)$, $xe^{-2x}\cos(x)$, and $xe^{-2x}\sin(x)$. These six solutions are linearly independent, so a general solution is

$$c_1e^{-3x} + c_2xe^{-3x} + c_3e^{-2x}\cos(x) + c_4e^{-2x}\sin(x) + c_5xe^{-2x}\cos(x) + c_6xe^{-2x}\sin(x) \ .$$

II. The function $\cos(x)$ satisfies the DE $y'' + y' + y = -\sin(x)$. Find a general solution.
(5)

The associated homogeneous DE is $y'' + y' + y = 0$. Its characteristic polynomial is $\lambda^2 + \lambda + 1$, which has roots $-1/2 \pm (\sqrt{3}/2)i$. Therefore a general solution of the associated homogeneous DE is

$$c_1e^{-x/2}\cos(\sqrt{3}x/2) + c_2e^{-x/2}\sin(\sqrt{3}x/2) \ ,$$

and a general solution of the original nonhomogeneous DE is

$$c_1e^{-x/2}\cos(\sqrt{3}x/2) + c_2e^{-x/2}\sin(\sqrt{3}x/2) + \cos(x) \ .$$

III. Show that the set of functions $\{1, 2\sin^2(x), \cos^2(x)\}$ is linearly dependent.
(3)

Since $\sin^2(x) + \cos^2(x) = 1$, we have $(1/2) \cdot 2\sin^2(x) + 1 \cdot \cos^2(x) + (-1) \cdot 1 = 0$. Since there is a nonzero linear combination of these functions that equals the zero function, the set is linearly dependent.

IV. Define what it means to say that a collection of functions $\{y_1, y_2, \ldots, y_n\}$ is *linearly independent.*
(3)

It means that $c_1y_1 + c_2y_2 + \cdots + c_ny_n = 0$ for constants c_i only when all the c_i are 0.