Mathematics 3113-005 Name (please print) ____________________

Quiz 5 Form B

March 11, 2011

Instructions: Give concise answers, but clearly indicate your reasoning.

I. (3) Define what it means to say that a collection of functions $\{y_1, y_2, \dots, y_n\}$ is *linearly independent.*

It means that $c_1y_1 + c_2y_2 + \cdots + c_ny_n = 0$ for constants c_i only when all the c_i are 0.
or
It means that if $c_1y_1 + c_2y_2 + \cdots + c_ny_n = 0$ for constants c_i, then all the c_i are 0.
or any of various other ways of stating this.

II. (7) Write trial solutions for using the method of undetermined coefficients to find a particular solution of the following DE's, but *do not* carry out the calculations or proceed further with obtaining a particular solution.

(a) $y'' + 9y = e^{3x}$

The associated homogeneous DE $y'' + 9y = 0$ has characteristic polynomial $\lambda^2 + 9$ with roots $\lambda = \pm 3i$, so the complementary functions are $y_c = c_1\cos(3x) + c_2\sin(3x)$. The trial solution is $x^s(Ae^{3x})$, and $s = 0$ is sufficient to ensure that no term is a complementary function. So a trial solution is $y_p = Ae^{3x}$.

(b) $y'' - 9y = e^{3x}$

The associated homogeneous DE $y'' - 9y = 0$ has characteristic polynomial $\lambda^2 - 9$ with roots $\lambda = \pm 3$, so the complementary functions are $y_c = c_1e^{3x} + c_2e^{-3x}$. The trial solution is $x^s(Ae^{3x})$, and $s = 1$ is sufficient to ensure that no term is a complementary function. So a trial solution is $y_p = Axe^{3x}$.

(c) $y^{(4)} + 6y'' + 9y = \cos(3x)$

The associated homogeneous DE $y^{(4)} + 6y'' + 9y = 0$ has characteristic polynomial $\lambda^4 + 6\lambda^2 + 9 = (\lambda^2+3)^2$, so has roots $\lambda = \pm\sqrt{3}i, \pm\sqrt{3}i$. Therefore the complementary functions are $y_c = c_1\cos(\sqrt{3}x) + c_2x\cos(x) + c_3\sin(\sqrt{3}x) + c_4x\sin(\sqrt{3}x)$. The trial solution is $x^s(A\cos(3x) + B\sin(3x))$, and $s = 0$ is sufficient to ensure that no term is a complementary function. So a trial solution is $y_p = A\cos(3x) + B\sin(3x)$.

III. (2) A certain mass-spring system is modeled by the second-order equation $x'' + cx' + 7x = 0$, where c is the damping constant. Find the value of c that gives critical damping (that is, the value of c for which the system neither overdamped nor underdamped).

The characteristic polynomial is $\lambda^2 + c\lambda + 7$, which has roots $\dfrac{-c \pm \sqrt{c^2 - 28}}{2}$. Critical damping occurs when there are two equal real roots, that is, when $c^2 = 28$ or $c = \sqrt{28}$ (which equals $2\sqrt{7}$, but this simplification is optional).

IV. (3) Suppose that the function $-8\cos(x) - 3\sin(x)$ is rewritten in phase-angle form $C\cos(\omega t - \alpha)$ (do not try to carry this out, just suppose that someone did). Give the phase angle α as an expression that involves a value of the inverse tangent function (that is, as an expression containing a number of the form $\tan^{-1}$(something), not a decimal number. You do not need to evaluate it on a calculator.)

We have $(A, B) = (-8, -3)$, which lies in the third quadrant. Since the angle $\tan^{-1}(B/A) = \tan^{-1}(3/8)$ lies in the first quadrant, α is $\pi + \tan^{-1}(3/8)$.