Mathematics 3113-005 Name (please print) ____________________

Quiz 6 Form A

April 8, 2011

Instructions: Give concise answers, but clearly indicate your reasoning.

I. (2) Define what it means to say that a collection of vectors $\{X_1, X_2, \ldots, X_n\}$ is *linearly independent.*

II. (5) Let $A = \begin{bmatrix} 3t & -1 & 0 \\ 2 & 1 & -t \\ 1 & 5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 0 \\ 2-t & 1 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} -\cos(t) & 3 & 0 \end{bmatrix}$.

(a) Tell which of the following six products are defined (do not do any calculations, just tell which ones are defined): AB, BA, AC, CA, BC, CB.

(b) Calculate $\det(A)$.

III. (3) Write the second-order system $x'' - 5x + 3y = 0$, $y'' + 2x + y = 0$ as an equivalent system of first-order equations.

IV. (3) Write the system $x_1' = 8x_1 + tx_2 + \cos(t)$, $x_2' = x_2 - x_3$, $x_3' = t + 2tx_2 - x_3$ in matrix form $X' = PX + F$. Do not proceed further with solving the system, just rewrite the general form $X' = PX + F$ with X, P and F written as matrices with the correct dimensions and entries for this particular system.

V. (2) For the system $X' = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} X$, verify that $X = e^{-t} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ is a solution.

VI. (3) Bonus problem: Graph the hyperbola $x^2 - \dfrac{y^2}{2} = 1$, showing the numerical values of the intercepts and the equations of the asymptotes.