Mathematics 3113-005 Name (please print) ______________________

Quiz 6 Form A

April 8, 2011

Instructions: Give concise answers, but clearly indicate your reasoning.

I. (2) Define what it means to say that a collection of vectors $\{X_1, X_2, \ldots, X_n\}$ is *linearly independent.*

It means that $c_1X_1 + c_2X_2 + \cdots + c_nX_n = 0$ for constants c_i only when all the c_i are 0.
[or]
It means that if $c_1X_1 + c_2X_2 + \cdots + c_nX_n = 0$ for constants c_i, then all the c_i are 0.

II. (5) Let $A = \begin{bmatrix} 3t & -1 & 0 \\ 2 & 1 & -t \\ 1 & 5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 0 \\ 2-t & 1 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} -\cos(t) & 3 & 0 \end{bmatrix}$.

(a) Tell which of the following six products are defined (do not do any calculations, just tell which ones are defined): AB, BA, AC, CA, BC, CB.

BA and CA are defined, AB, AC, BC, and CB are not defined.

(b) Calculate $\det(A)$.

The easiest way is to expand down the third column of A, since that column contains two zeros. We find that

$$\det(A) = 0 - (-t) \cdot (3t \cdot 5 - (-1) \cdot 1) + 0 = 15t^2 + t\ .$$

III. (3) Write the second-order system $x'' - 5x + 3y = 0$, $y'' + 2x + y = 0$ as an equivalent system of first-order equations.

We define new functions x_1, x_2, y_1, and y_2 by $x_1 = x$, $x_2 = x'$, $y_1 = y$, and $y_2 = y'$. With these definitions, the second-order system becomes

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= 5x_1 - 3y_1 \\ y_1' &= y_2 \\ y_2' &= -2x_1 - y_1 \end{aligned}$$

IV. (3) Write the system $x_1' = 8x_1 + tx_2 + \cos(t)$, $x_2' = x_2 - x_3$, $x_3' = t + 2tx_2 - x_3$ in matrix form $X' = PX + F$. Do not proceed further with solving the system, just rewrite the general form $X' = PX + F$ with X, P and F written as matrices with the correct dimensions and entries for this particular system.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}' = \begin{bmatrix} 8 & t & 0 \\ 0 & 1 & -1 \\ 0 & 2t & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \cos(t) \\ 0 \\ t \end{bmatrix}$$

V. (2) For the system $X' = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} X$, verify that $X = e^{-t} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ is a solution.

We have $X' = -e^{-t} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, and we compute that

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot e^{-t} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = e^{-t} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = e^{-t} \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = -e^{-t} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = X' \, .$$

VI. (3) Bonus problem: Graph the hyperbola $x^2 - \dfrac{y^2}{2} = 1$, showing the numerical values of the intercepts and the equations of the asymptotes.