Math 4513 homework

Homework Assignment #1 (due 1/26). Prepare for a class discussion on the topic of "What is mathematics?". In our discussion, we will attempt to give a definition of mathematics. This is an extremely difficult problem, and it does not have a well-defined solution. Still, trying to come up with a good definition should deepen our understanding of our favorite subject.

Ideally, a definition should have the following properties:

(1) It should be reasonably clear and, at least in most cases, reasonably easy to apply to test whether or not something is mathematics.

(2) Any mathematics should meet the conditions of the definition.

(3) Anything that is not mathematics should fail to meet the conditions of the definition.

(4) It should be short, and as simple as possible.

Have something written down, and if you can, have some specific examples on which you can apply your definition, both verifying that something is mathematics and verifying that something is not. Be prepared to stand at the board and lead a discussion on your definition. You could also be ready with more than one candidate for a definition, why not?

Here is an example of a definition that does not work well: Something is mathematics if it uses numbers or quantities to solve problems.

This satisfies (1) and (4). But it doesn't really work very well on (2), since lots of mathematics involves more abstract ideas that may use numbers only incidentally if at all. And lots of mathematics is a matter of building theories or establishing mathematical facts, not really "solving problems." And in my opinion the definition fails completely on (3), since numbers are used to solve of problems in business, chemistry, and so on. If we call every such use of numbers mathematics, we haven't really achieved any understanding of why we consider ourselves mathematicians, and those other folks not mathematicians. Our definition should try to get at the heart of the matter— for example, what makes a mathematician different from a physicist?

This is a very challenging assignment— indeed, the best philosophers of mathematics cannot reach agreement on it— and if it makes you feel somewhat helpless, that might just mean that you appreciate the difficulty. Just do your best, and we should be able to have a useful discussion.

Homework Assignment #2 (due 2/11). Find examples of the ideas and methods that are discussed in my seminars on mathematical proof. Good sources would be your linear algebra textbook, perhaps your calculus textbook, and textbooks from other courses that you have had. Look for simple, relatively straightforward and elementary examples— the purpose is not to be fancy, but rather to find simple, clear, memorably beautiful (what mathematicians call "elegant") examples that we can carry with us as proof prototypes in our mathematical thinking. Start your own evolving list of "favorite proofs". For example, two of those on my list are a proof that the internal angles of a triangle add up to π, and a proof that there are infinitely many prime numbers. Be prepared to present any of your examples in class (including giving definitions and explanations of terms that others might not be familar with). Perhaps we can develop a class ist of favorite examples illustrating most of the different methods that will be discussed, such as direct proof, proving the contrapositive, and proof by contradiction.