Math 1823-004 Bonus Quiz Spring 2016

Name: Solution

Student Number:

Problem 1
Compute

$$\lim_{\theta\to 0}\frac{\sin(3\theta)}{\sin(7\theta)} = \lim_{\theta\to 0}\frac{\sin(3\theta)}{1}\cdot\frac{1}{\sin(7\theta)}$$

$$= \lim_{\theta\to 0}\frac{\sin(3\theta)}{3\theta}\cdot 3\theta\cdot\frac{7\theta}{\sin(7\theta)}\cdot\frac{1}{7\theta}$$

$$= \frac{3}{7}\cdot\lim_{\theta\to 0}\frac{\sin(3\theta)}{3\theta}\cdot\lim_{\theta\to 0}\frac{7\theta}{\sin(7\theta)}$$

$$= \frac{3}{7}\cdot 1\cdot 1 = \frac{3}{7}.$$

Problem 2
Write the **mathematical** definition of the derivative $f'(a)$ of the function f at $x = a$.

$$f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$$= \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

Problem 3
Use your knowledge of derivatives to find the numerical value of the limit

$$\lim_{x\to\pi/6}\frac{\sin x - 1/2}{x-\pi/6}$$

Write one sentence to explain how you obtained your answer.

Compare $\lim_{x\to\frac{\pi}{6}}\frac{\sin x-\frac{1}{2}}{x-\frac{\pi}{6}} = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

Choose $f(x)=\sin x$ and $a=\frac{\pi}{6}$

Then, $f(a)=\sin\frac{\pi}{6}=\frac{1}{2}$ ✓

So, $\lim_{x\to\frac{\pi}{6}}\frac{\sin x-\frac{1}{2}}{x-\frac{\pi}{6}} = f'(\frac{\pi}{6})$ but $f'(x)=\cos x$

So, $f'(\frac{\pi}{6})=\frac{\sqrt{3}}{2}$

$$\lim_{x\to\pi/6}\frac{\sin x-\frac{1}{2}}{x-\frac{\pi}{6}} = \frac{\sqrt{3}}{2}$$

Problem 4

Find $\frac{dy}{dx}$ by implicit differentiation.

$$2x^2 + xy - y^2 = 2$$

Differentiate both sides w.r.t. x:

$$4x + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$$

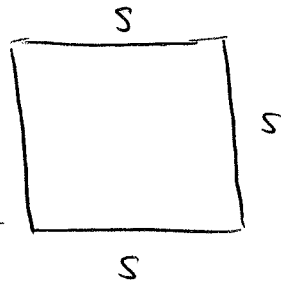
$$\frac{dy}{dx}[x - 2y] = -4x - y$$

$$\frac{dy}{dx} = \frac{-4x - y}{x - 2y}$$

Problem 5

Each side s of a square is increasing at a rate of 6 cm/s. At what rate is the area A of the square increasing when the area of the square is 16 cm^2.

Step 1:



Step 2: s : side of square

A : Area of square

Step 3: Given: $\frac{ds}{dt} = 6$ cm/s

Unknown: $\frac{dA}{dt} = ?$ when $A = 16$ cm^2

Step 4: $A = s^2$

$$\frac{dA}{dt} = 2s\frac{ds}{dt}$$

For $A = 16$ cm^2, $s = 4$ cm

So, $\frac{dA}{dt} = 2 \cdot (4)(6) = 48$ cm^2/s