Math 1823-004 Quiz 14 Spring 2016

Name: Solution

Student Number:

Problem 1
Find the horizontal and vertical asymptotes of the function

$$f(x) = \frac{x^2+3x+2}{x^2+4x+3}$$

Horizontal Asymptotes:

(1) $\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{x^2+3x+2}{x^2+4x+3}$

$= \lim_{x\to\infty} \frac{\frac{x^2+3x+2}{x^2}}{\frac{x^2+4x+3}{x^2}}$

$= \lim_{x\to\infty} \frac{1+\frac{3}{x}+\frac{2}{x^2}}{1+\frac{4}{x}+\frac{3}{x^2}} = \frac{1+0+0}{1+0+0} = 1.$

(2) $\lim_{x\to-\infty} f(x) = 1$

↑ You can check this in a similar manner

So, $y=1$ is a H.A.

Vertical Asymptotes:

Since $x=-1$ & $x=-3$ make the denominator zero, we need to check $\lim_{x\to-1} f(x)$ & $\lim_{x\to-3} f(x)$.

(1) $\lim_{x\to-1} \frac{x^2+3x+2}{x^2+4x+3} = \lim_{x\to-1} \frac{(x+1)(x+2)}{(x+1)(x+3)} = \lim_{x\to-1} \frac{x+2}{x+3}$

$= \frac{-1+2}{-1+3} = \frac{1}{2} \neq \pm\infty$

Hence $x=-1$ is NOT a vertical asymptote

(2) $\lim_{x\to-3} \frac{x^2+3x+2}{x^2+4x+3} = \lim_{x\to-3} \frac{x+2}{x+3}$

$\lim_{x\to-3^-} \frac{x+2}{x+3} = +\infty$ $\lim_{x\to-3^+} \frac{x+2}{x+3} = -\infty$

So, $x=-3$ is a V.A.

NOTE: We had to check sided limits in seperate cases.