

Name: Solution

Student Number:

Problem 1

Find the derivative of the following functions using the definition of derivative.

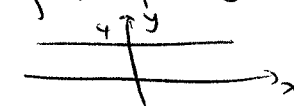
Recall: The derivative of a function $y = f(x)$ is defined by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

(a) $f(x) = 4$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{4 - 4}{h} = 0 \end{aligned}$$

Remember:
 $f(x) = 4$ has a graph



$f'(x) =$ slope of tangent line
 but tangent line at every
 pt. is the same horizontal
 line and so slope is 0
 (horizontal lines have
 slope 0)

(b) $f(x) = x^2$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(2x+h)}{h} = \lim_{h \rightarrow 0} (2x+h) = 2x \end{aligned}$$

(c) $f(x) = 1/x$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{x - (x+h)}{h \cdot x \cdot (x+h)} \\ &= \lim_{h \rightarrow 0} \frac{-h}{h \cdot x \cdot (x+h)} = \lim_{h \rightarrow 0} \frac{-1}{x \cdot (x+h)} = -\frac{1}{x^2} \end{aligned}$$