13. Describe several ways in which a limit can fail to exist. Illustrate with sketches.

14. What does it mean to say that the line $x = a$ is a vertical asymptote of the curve $y = f(x)$? Draw curves to illustrate the various possibilities.

15. State the following Limit Laws.
    (a) Sum Law
    (b) Difference Law
    (c) Constant Multiple Law
    (d) Product Law
    (e) Quotient Law
    (f) Power Law
    (g) Root Law

16. What does the Squeeze Theorem say?

17. (a) What does it mean for f to be continuous at a?
    (b) What does it mean for f to be continuous on the interval $(-\infty, \infty)$? What can you say about the graph of such a function?

18. (a) Give examples of functions that are continuous on $[-1, 1]$.
    (b) Give an example of a function that is not continuous on $[0, 1]$.

19. What does the Intermediate Value Theorem say?

TRUE-FALSE QUIZ

Determine whether the statement is true or false. If it is true, explain why. If it is false, explain why or give an example that disproves the statement.

1. If f is a function, then $f(s + t) = f(s) + f(t)$.

2. If $f(s) = f(t)$, then $s = t$.

3. If f is a function, then $f(3x) = 3f(x)$.

4. If $x_1 < x_2$ and f is a decreasing function, then $f(x_1) > f(x_2)$.

5. A vertical line intersects the graph of a function at most once.

6. If x is any real number, then $\sqrt{x^2} = x$.

7. $\lim_{x\to 4}\left(\dfrac{2x}{x-4} - \dfrac{8}{x-4}\right) = \lim_{x\to 4}\dfrac{2x}{x-4} - \lim_{x\to 4}\dfrac{8}{x-4}$

8. $\lim_{x\to 1}\dfrac{x^2+6x-7}{x^2+5x-6} = \dfrac{\lim_{x\to 1}(x^2+6x-7)}{\lim_{x\to 1}(x^2+5x-6)}$

9. $\lim_{x\to 1}\dfrac{x-3}{x^2+2x-4} = \dfrac{\lim_{x\to 1}(x-3)}{\lim_{x\to 1}(x^2+2x-4)}$

10. $\dfrac{x^2-9}{x-3} = x + 3$

11. $\lim_{x\to 3}\dfrac{x^2-9}{x-3} = \lim_{x\to 3}(x+3)$

12. If $\lim_{x\to 5} f(x) = 2$ and $\lim_{x\to 5} g(x) = 0$, then $\lim_{x\to 5}[f(x)/g(x)]$ does not exist.

13. If $\lim_{x\to 5} f(x) = 0$ and $\lim_{x\to 5} g(x) = 0$, then $\lim_{x\to 5}[f(x)/g(x)]$ does not exist.

14. If neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exists, then $\lim_{x\to a}[f(x) + g(x)]$ does not exist.

15. If $\lim_{x\to a} f(x)$ exists but $\lim_{x\to a} g(x)$ does not exist, then $\lim_{x\to a}[f(x) + g(x)]$ does not exist.

16. If $\lim_{x\to 6}[f(x)\,g(x)]$ exists, then the limit must be $f(6)\,g(6)$.

17. If p is a polynomial, then $\lim_{x\to b} p(x) = p(b)$.

18. If $\lim_{x\to 0} f(x) = \infty$ and $\lim_{x\to 0} g(x) = \infty$, then $\lim_{x\to 0}[f(x) - g(x)] = 0$.

19. If the line $x = 1$ is a vertical asymptote of $y = f(x)$, then f is not defined at 1.

20. If $f(1) > 0$ and $f(3) < 0$, then there exists a number c between 1 and 3 such that $f(c) = 0$.

21. If f is continuous at 5 and $f(5) = 2$ and $f(4) = 3$, then $\lim_{x\to 2} f(4x^2 - 11) = 2$.

22. If f is continuous on $[-1, 1]$ and $f(-1) = 4$ and $f(1) = 3$, then there exists a number r such that $|r| < 1$ and $f(r) = \pi$.

23. Let f be a function such that $\lim_{x\to 0} f(x) = 6$. Then there exists a positive number δ such that if $0 < |x| < \delta$, then $|f(x) - 6| < 1$.

24. If $f(x) > 1$ for all x and $\lim_{x\to 0} f(x)$ exists, then $\lim_{x\to 0} f(x) > 1$.

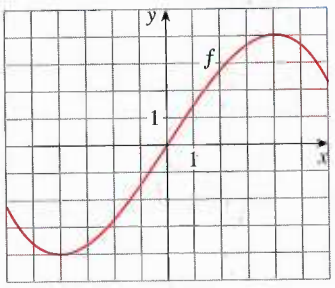
25. The equation $x^{10} - 10x^2 + 5 = 0$ has a root in the interval $(0, 2)$.

26. If f is continuous at a, so is $|f|$.

27. If $|f|$ is continuous at a, so is f.

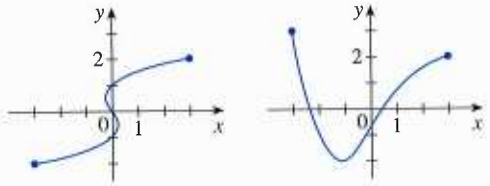
EXERCISES

1. Let f be the function whose graph is given.



(a) Estimate the value of $f(2)$.
(b) Estimate the values of x such that $f(x) = 3$.
(c) State the domain of f.
(d) State the range of f.
(e) On what interval is f increasing?
(f) Is f even, odd, or neither even nor odd? Explain.

2. Determine whether each curve is the graph of a function of x. If it is, state the domain and range of the function.



3. If $f(x) = x^2 - 2x + 3$, evaluate the difference quotient

$$\frac{f(a+h) - f(a)}{h}$$

4. Sketch a rough graph of the yield of a crop as a function of the amount of fertilizer used.

5–8 Find the domain and range of the function. Write your answer in interval notation.

5. $f(x) = 2/(3x - 1)$

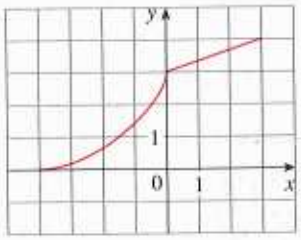
6. $g(x) = \sqrt{16 - x^4}$

7. $y = 1 + \sin x$

8. $F(t) = 3 + \cos 2t$

9. Suppose that the graph of f is given. Describe how the graphs of the following functions can be obtained from the graph of f.
(a) $y = f(x) + 8$
(b) $y = f(x + 8)$
(c) $y = 1 + 2f(x)$
(d) $y = f(x - 2) - 2$
(e) $y = -f(x)$
(f) $y = 3 - f(x)$

10. The graph of f is given. Draw the graphs of the following functions.
(a) $y = f(x - 8)$
(b) $y = -f(x)$
(c) $y = 2 - f(x)$
(d) $y = \frac{1}{2}f(x) - 1$



11–16 Use transformations to sketch the graph of the function.

11. $y = (x - 2)^3$

12. $y = 2\sqrt{x}$

13. $y = x^2 - 2x + 2$

14. $y = \dfrac{1}{x - 1}$

15. $f(x) = -\cos 2x$

16. $f(x) = \begin{cases} 1 + x & \text{if } x < 0 \\ 1 + x^2 & \text{if } x \geqslant 0 \end{cases}$

17. Determine whether f is even, odd, or neither even nor odd.
(a) $f(x) = 2x^5 - 3x^2 + 2$
(b) $f(x) = x^3 - x^7$
(c) $f(x) = \cos(x^2)$
(d) $f(x) = 1 + \sin x$

18. Find an expression for the function whose graph consists of the line segment from the point $(-2, 2)$ to the point $(-1, 0)$ together with the top half of the circle with center the origin and radius 1.

19. If $f(x) = \sqrt{x}$ and $g(x) = \sin x$, find the functions (a) $f \circ g$, (b) $g \circ f$, (c) $f \circ f$, (d) $g \circ g$, and their domains.

20. Express the function $F(x) = 1/\sqrt{x + \sqrt{x}}$ as a composition of three functions.

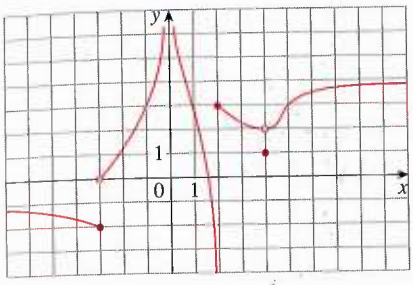
21. Life expectancy improved dramatically in the 20th century. The table gives the life expectancy at birth (in years) of males born in the United States. Use a scatter plot to choose an appropriate type of model. Use your model to predict the life span of a male born in the year 2010.

Birth year	Life expectancy	Birth year	Life expectancy
1900	48.3	1960	66.6
1910	51.1	1970	67.1
1920	55.2	1980	70.0
1930	57.4	1990	71.8
1940	62.5	2000	73.0
1950	65.6		

22. A small-appliance manufacturer finds that it costs \$9000 to produce 1000 toaster ovens a week and \$12,000 to produce 1500 toaster ovens a week.
(a) Express the cost as a function of the number of toaster ovens produced, assuming that it is linear. Then sketch the graph.

(b) What is the slope of the graph and what does it represent?
(c) What is the y-intercept of the graph and what does it represent?

23. The graph of f is given.



(a) Find each limit, or explain why it does not exist.
(i) $\lim_{x\to 2^+} f(x)$ (ii) $\lim_{x\to -3^+} f(x)$ (iii) $\lim_{x\to -3} f(x)$
(iv) $\lim_{x\to 4} f(x)$ (v) $\lim_{x\to 0} f(x)$ (vi) $\lim_{x\to 2^-} f(x)$
(b) State the equations of the vertical asymptotes.
(c) At what numbers is f discontinuous? Explain.

24. Sketch the graph of an example of a function f that satisfies all of the following conditions:
$\lim_{x\to 0^+} f(x) = -2$, $\lim_{x\to 0^-} f(x) = 1$, $f(0) = -1$,
$\lim_{x\to 2^-} f(x) = \infty$, $\lim_{x\to 2^+} f(x) = -\infty$

25–38 Find the limit.

25. $\lim_{x\to 0} \cos(x + \sin x)$

26. $\lim_{x\to 3} \dfrac{x^2 - 9}{x^2 + 2x - 3}$

27. $\lim_{x\to -3} \dfrac{x^2 - 9}{x^2 + 2x - 3}$

28. $\lim_{x\to 1^+} \dfrac{x^2 - 9}{x^2 + 2x - 3}$

29. $\lim_{h\to 0} \dfrac{(h-1)^3 + 1}{h}$

30. $\lim_{t\to 2} \dfrac{t^2 - 4}{t^3 - 8}$

31. $\lim_{r\to 9} \dfrac{\sqrt{r}}{(r-9)^4}$

32. $\lim_{v\to 4^+} \dfrac{4 - v}{|4 - v|}$

33. $\lim_{u\to 1} \dfrac{u^4 - 1}{u^3 + 5u^2 - 6u}$

34. $\lim_{x\to 3} \dfrac{\sqrt{x+6} - x}{x^3 - 3x^2}$

35. $\lim_{s\to 16} \dfrac{4 - \sqrt{s}}{s - 16}$

36. $\lim_{v\to 2} \dfrac{v^2 + 2v - 8}{v^4 - 16}$

37. $\lim_{x\to 0} \dfrac{1 - \sqrt{1 - x^2}}{x}$

38. $\lim_{x\to 1} \left(\dfrac{1}{x-1} + \dfrac{1}{x^2 - 3x + 2} \right)$

39. If $2x - 1 \le f(x) \le x^2$ for $0 < x < 3$, find $\lim_{x\to 1} f(x)$.

40. Prove that $\lim_{x\to 0} x^2 \cos(1/x^2) = 0$.

41–44 Prove the statement using the precise definition of a limit.

41. $\lim_{x\to 2} (14 - 5x) = 4$

42. $\lim_{x\to 0} \sqrt[3]{x} = 0$

43. $\lim_{x\to 2} (x^2 - 3x) = -2$

44. $\lim_{x\to 4^+} \dfrac{2}{\sqrt{x-4}} = \infty$

45. Let

$$f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \le x < 3 \\ (x-3)^2 & \text{if } x > 3 \end{cases}$$

(a) Evaluate each limit, if it exists.
(i) $\lim_{x\to 0^+} f(x)$ (ii) $\lim_{x\to 0^-} f(x)$ (iii) $\lim_{x\to 0} f(x)$
(iv) $\lim_{x\to 3^-} f(x)$ (v) $\lim_{x\to 3^+} f(x)$ (vi) $\lim_{x\to 3} f(x)$
(b) Where is f discontinuous?
(c) Sketch the graph of f.

46. Let

$$g(x) = \begin{cases} 2x - x^2 & \text{if } 0 \le x \le 2 \\ 2 - x & \text{if } 2 < x \le 3 \\ x - 4 & \text{if } 3 < x < 4 \\ \pi & \text{if } x \ge 4 \end{cases}$$

(a) For each of the numbers 2, 3, and 4, discover whether g is continuous from the left, continuous from the right, or continuous at the number.
(b) Sketch the graph of g.

47–48 Show that the function is continuous on its domain. State the domain.

47. $h(x) = \sqrt[4]{x} + x^3 \cos x$

48. $g(x) = \dfrac{\sqrt{x^2 - 9}}{x^2 - 2}$

49–50 Use the Intermediate Value Theorem to show that there is a root of the equation in the given interval.

49. $x^5 - x^3 + 3x - 5 = 0$, $(1, 2)$

50. $2 \sin x = 3 - 2x$, $(0, 1)$

51. Suppose that $|f(x)| \le g(x)$ for all x, where $\lim_{x\to a} g(x) = 0$. Find $\lim_{x\to a} f(x)$.

52. Let $f(x) = [\![x]\!] + [\![-x]\!]$.
(a) For what values of a does $\lim_{x\to a} f(x)$ exist?
(b) At what numbers is f discontinuous?