Worksheet 11 - Section 2.6

(1) (a) Find y' by implicit differentiation.

(b) Solve the equation explicitly for y and differentiate to get y' in terms of x.

(c) Check that your solutions to parts (a) and (b) are consistent by substituting the expression for y into your solution for part (a).

$$\frac{1}{x} + \frac{1}{y} = 1$$

(2) Find $\dfrac{dy}{dx}$ by implicit differentiation.

(a) $y^5 + x^2y^3 = 1 + x^4y$

(b) $y\cos x = x^2 + y^2$

(c) $4\cos x \sin y = 1$

(d) $\sqrt{x+y} = 1 + x^2y^2$

(e) $\cos(x+y) = \sin(xy)$

(f) $\tan(\frac{x}{y}) = x + y$

(g) $\dfrac{x^2}{x+y} = y^2 + 1$

(h) $xy = \sqrt{x^2+y^2}$

(3) If $f(x) + x^2[f(x)]^3 = 10$ and $f(1) = 2$, , find $f'(1)$.

(4) Regard y as the independent variable and x as the dependent variable and use implicit differentiation to find $\frac{dy}{dx}$.

$$x^4y^2 - x^3y + 2xy^3 = 0$$

(5) Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

(a) $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$ at $(0, \frac{1}{2})$.

(b) $y^2(y^2 - 4) = x^2(x^2 - 5)$ at $(0, -2)$.

(6) Find y'' by implicit differentiation.

(a) $\sqrt{x} + \sqrt{y} = 1$

(b) $\sin y + \cos x = 1$

(7) If $xy + y^3 = 1$, find the value of y'' at the point where $x = 0$.