Math 1823 - Section 004 **Spring 2016**

Worksheet 5 - Section 1.8

(1) Sketch the graph of a function that is continuous except for the stated discontinuity.

(a) Removable discontinuity at 3 and jump discontinuity at 5.

(b) Discontinuous, but cntinuous from the right at 2.

(2) Use the definition of continuity and the properties of limits to show that the function is continuous on the given interval.

$$f(x) = \frac{2x+3}{x-2}, \quad (2, \infty)$$

(3) Explain why the function is discontinuous at the given number a. Sketch the graph of the function.

(a) $f(x) = \dfrac{1}{x+2}, \quad a = -2$

(b) $f(x) = \begin{cases} 1 - x^2, & \text{if } x < 1 \\ \frac{1}{x}, & \text{if } x \geq 1 \end{cases} \qquad a = 1$

(c) $f(x) = \begin{cases} \cos x, & \text{if } x < 0 \\ 0, & \text{if } x = 0 \\ 1 - x^2, & \text{if } x > 0 \end{cases} \qquad a = 0$

(4) How would you "remove the discontinuity" of f? In other words, how would you define $f(2)$ in order to make f continuous at 2?

$$f(x) = \frac{x^2 - x - 2}{x - 2}$$

(5) Find the numbers at which f is discontinuous. At which of these numbers is f continuous from the right, left, or neither? Sketch the graph of f.

$$f(x) = \begin{cases} x + 1, & x \leq 1 \\ \frac{1}{x}, & 1 < x < 3 \\ \sqrt{x-3}, & x \geq 3 \end{cases}$$

(6) Use continuity to evaluate the limit.

(a) $\lim_{x \to 4} \frac{5+\sqrt{x}}{\sqrt{5+x}}$ (b) $\lim_{x \to 2} (x^3 - 3x + 1)^{-3}$