Worksheet 8 - Section 2.3

(1) Differentiate the function.

(a) $f(x) = 2^{40}$ (b) $g(t) = 2t^{-\frac{3}{4}}$

(c) $y = x^2(1-2x)$ (d) $y = \sqrt{x}(x-1)$

(e) $S(R) = 4\pi R^2$ (f) $y = \frac{x^2+4x+3}{\sqrt{x}}$

(g) $H(x) = (x+x^{-1})^3$ (h) $G(q) = (\frac{1}{t} - \frac{1}{\sqrt{t}})^2$

(i) $F(y) = (\frac{1}{y^2} - \frac{3}{y^4})(y+5y^3)$ (j) $y = \frac{t^3+3t}{t^2-4t+3}$

(k) $g(x) = \frac{1+2x}{3-4x}$ (l) $g(t) = \frac{t-\sqrt{t}}{t^{\frac{1}{3}}}$

(2) Find the derivative of $f(x) = (1+2x^2)(x-x^2)$ in two ways: by using the Product Rule and by performing the multiplication first. Do your answers agree?

(3) Find equations of the tangent line and normal line to the curve

$$y = \frac{3x+1}{x^2+1}$$

at the point $(1, 2)$.

(4) Find the first and second derivatives of the function

$$f(x) = \frac{x^2}{1+2x}$$

(5) The equation of motion of a particle is $s = t^3 - 3t$, where s is in meters and t is in seconds. Find

(a) the velocity and acceleration as a function of t,

(b) the acceleration after 2 s, and

(c) the acceleration when the velocity is 0.

(6) Suppose that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$ and $g'(5) = 2$.
Find the following values:

(a) $(fg)'(5)$, (b) $(\frac{f}{g})'(5)$ (c) $(\frac{g}{f})'(5)$

(7) If $f(x) = \sqrt{x}g(x)$ where $g(4) = 8$ and $g'(4) = 7$, find $f'(4)$.

(8) If g is a differentiable function, find an expression for the derivative of each of the following functions:
(a) $y = xg(x)$ (b) $y = \frac{x}{g(x)}$ (c) $y = \frac{g(x)}{x}$

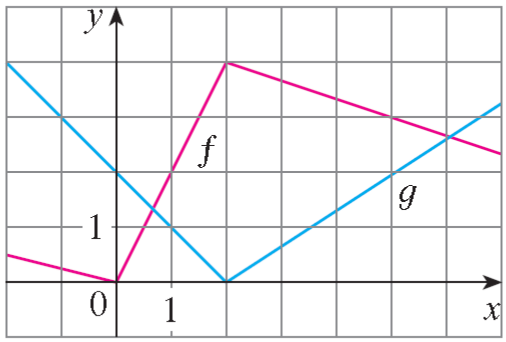
(9) Show that the curve $y = 6x^3 + 5x - 3$ has no tangent line with slope 4.

(10) Find an equation of the normal line to the curve $y = \sqrt{x}$ that is parallel to the line $2x + y = 1$.

(11) If f and g are functions whose graphs are shown, let

$$u(x) = f(x)g(x) \text{ and } v(x) = \frac{f(x)}{g(x)}.$$

(a) Find $u'(1)$. (b) Find $v'(5)$



(12) Where does the normal line to the parabola $y = x^2 - 1$ at the point $(-1, 0)$ intersect the parabola a second time? Illustrate with a sketch.

(13) Let

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1 \\ x + 1 & \text{if } x \geq 1 \end{cases}$$

Is f differentiable at 1? Sketch the graphs of f and f'.

(14) Let

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ mx + b & \text{if } x > 2 \end{cases}$$

Find the vaues of m and b that make f differentiable everywhere.