

Worksheet 9 - Section 2.4

(1) Differentiate.

(a) $y = \sec \theta \tan \theta$

(b) $f(\theta) = \frac{\sec \theta}{1 + \sec \theta}$

(c) $h(\theta) = \theta \csc \theta - \cot \theta$

(d) $y = x^2 \sin x \tan x$

(e) $y = \frac{t \sin t}{1 + t}$

(f) $f(\theta) = \theta \cos \theta \sin \theta$

(2) Prove that

$$\begin{aligned}\frac{d}{dx}(\csc x) &= -\csc x \cot x \\ \frac{d}{dx}(\cot x) &= -\csc^2 x\end{aligned}$$

(3) Find an equation of the tangent line to the curve

$$y = \cos x - \sin x$$

at the point $(\pi, -1)$.

(4) If $H(\theta) = \theta \sin \theta$, find $H'(\theta)$ and $H''(\theta)$.

(5) (a) Use the Quotient Rule to differentiate the function

$$f(x) = \frac{\tan x - 1}{\sec x}$$

(b) Simplify the expression for $f(x)$ by writing it in terms of $\sin x$ and $\cos x$, and then find $f'(x)$.

(c) Show that your answers to parts (a) and (b) are equivalent.

(6) For what values of x does the graph of $f(x) = x + 2 \sin x$ have a horizontal tangent?

(7) Find the limit.

(a) $\lim_{t \rightarrow 0} \frac{\tan(6t)}{\sin(2t)}$

(b) $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta + \tan \theta}$

(c) $\lim_{x \rightarrow 0} \frac{\sin(3x)}{5x^3 - 4x}$

(d) $\lim_{x \rightarrow \pi/4} \frac{1 - \tan x}{\sin x - \cos x}$

(e) $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$

(f) $\lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right)$