

Name:

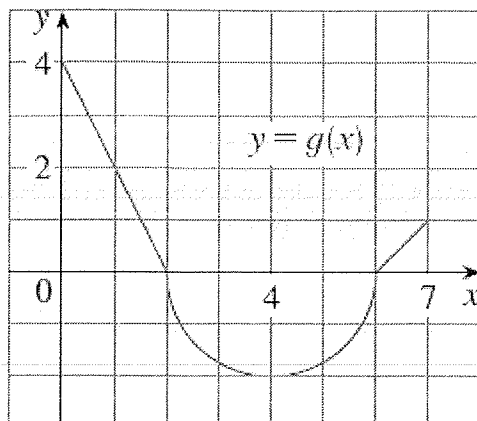
Student Number:

Show all work and answers on these sheets. You are not allowed to use a calculator or a formula/reference sheet. *Explain briefly your reasoning.* **Good luck!**



Problem 1. [2+2+2 points]

The graph of g consists of two straight lines and a semicircle as shown in the figure below.



Use the above graph to evaluate each integral.

(a) $\int_0^2 g(x) dx$

(b) $\int_2^6 g(x) dx$

(c) $\int_0^7 g(x) dx$

Problem 2. [6 points]

Let $g(x) = 1 + x$. Evaluate the definite integral

$$\int_{-1}^1 g(x) dx$$

using the definition of a definite integral as a limit of Riemann sums.

(Here are the sum formulas we used in class, in case you need them:

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^n i^3 = \left[\frac{n(n+1)}{2} \right]^2.)$$

Problem 3. [2+3 points]

Let $F(x) = \int_{\pi}^x \frac{\cos t}{t} dt$.

(a) Find the derivative of $F(x)$.

(b) Use part (a) to find an equation of the tangent line to the curve $y = F(x)$ at the point with x-coordinate π .

Problem 4. [2 points]

If $f(1) = 12$, f' is continuous and $\int_1^4 f'(x) dx = 17$, what is the value of $f(4)$?
Make sure you show all your work!

Problem 5. [2+2+3+2+3+3 points]

Evaluate the following indefinite and definite integrals.

(a) $\int (\pi + 5 \sec^2 \theta) d\theta$

(b) $\int \frac{1 + \sqrt{x} + x}{\sqrt{x}} dx$

(c) $\int x \sqrt{1 - x^2} dx$

$$(d) \int_0^1 (1 - 8v^3 + 16v^7) dv$$

$$(e) \int_0^{\pi/2} \cos x \sin(\sin x) dx$$

$$(f) \int_{-3}^0 \sqrt{9 - x^2} dx \quad (\text{Hint: Interpret the integral in terms of areas.})$$

Problem 6. [1+3+2 points]

In this problem, you'll find the area of the region enclosed by the curves $y = x$ and $y = (x - 2)^2$.

(a) Sketch the curves $y = x$ and $y = (x - 2)^2$ and shade the region enclosed by the curves.

(b) Set up the definite integral that gives the area of the region enclosed by the curves.

(c) Evaluate the definite integral from part (b) to find the area of the region.

Bonus [2 points]

Evaluate the following integral.

$$\int x^5 \sqrt{x^2 + 1} \, dx$$

The End!