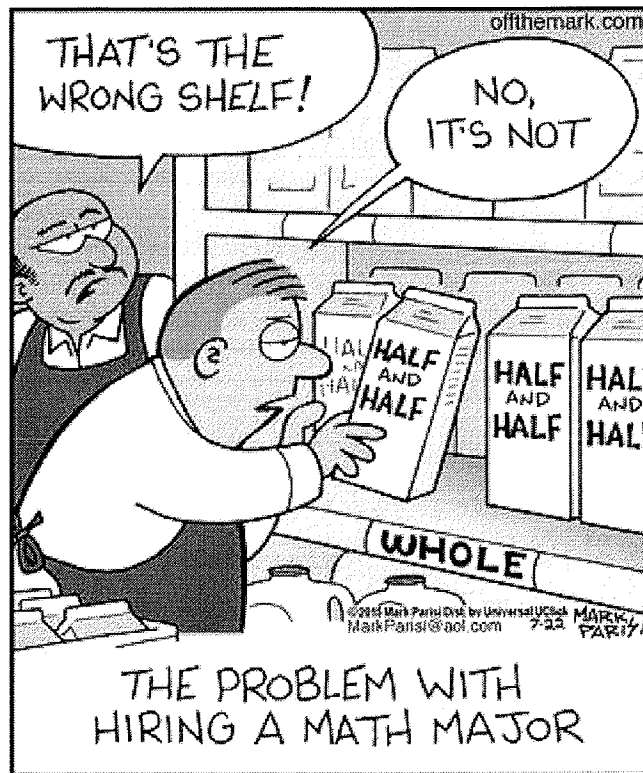


Name:

Student Number:

Show all work and answers on these sheets. You are not allowed to use a calculator or a formula/reference sheet. *Explain briefly your reasoning.* Good luck!



Problem 1. [3+2 points]

Given the function $F(x) = \ln(1 + x^2)$.

(a) Find the intervals over which $f(x)$ is increasing or decreasing.

(b) Find an equation of the tangent line to the curve of $y = F(x)$ at the point $(1, \ln 2)$.

Problem 2. [1+4+2 points]

Let R be the region bounded between $f(x) = 4 - x^2$, $x = 0$ and $y = 0$. Let S be the solid obtained by rotating R about the line $x = 5$.

(a) Draw a rough sketch of the region R , the axis of rotation and the solid S obtained.

(b) Set up an integral for the volume of the solid S while indicating which method you're using.

(c) Evaluate the integral from part (b) to find the volume of the solid S .

Problem 3. [3 points]

Let $f(x) = 2x \sin x + e^x$. If we restrict the domain of $f(x)$ to a small interval containing $x = 0$, then the function $f(x)$ becomes one-to-one on that interval. Let $g(x)$ be the inverse of $f(x)$ on this small interval. Compute $g'(1)$. **Make sure you show all your work!**

Problem 4. [3 points]

Let $h(x) = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$. Find the average value of the function $h(x)$ over the interval $[4, 9]$.

Problem 5. [3+3+3+3 points]

Evaluate the following indefinite and definite integrals. **Show all your work!**

(a) $\int \sin \theta e^{5+3 \cos \theta} d\theta$

(b) $\int \frac{3}{1+25x^2} dx$

$$(c) \int_0^1 \frac{(1 + 2^x)^2}{2^x} dx$$

$$(d) \int_0^{1/2} \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$$

Problem 6. [4 points]

Find the derivative of the function

$$y = \frac{(x+1)^4(x-5)^3}{(x-3)^8}$$

Hint: You can use logarithmic differentiation.

Problem 7. [3+3 points]

Evaluate the following limits. Show all your work!

(a) $\lim_{x \rightarrow \frac{1}{2}} \frac{6x^2 + 5x - 4}{4x^2 + 16x - 9}$

(b) $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{\cos x + e^x - 1}$

Bonus [2 points]

Find an equation of the tangent line to the curve $x^y = y^x$ at the point $(1,1)$.

The End!

