

Name: *Solution*

Student Number:

Problem 1

In this problem, you will evaluate the integral

$$\int \frac{\sec^2 t}{\tan^2 t + 3 \tan t + 2} dt$$

(a) Make a substitution to express the integral as a rational function.

$$\begin{aligned} \text{let } u &= \tan t \\ du &= \sec^2 t dt \end{aligned}$$

$$\int \frac{\sec^2 t}{\tan^2 t + 3 \tan t + 2} dt = \int \frac{du}{u^2 + 3u + 2}$$

(b) Evaluate the resulting integral from part (a) using partial fractions.

$$\int \frac{du}{u^2 + 3u + 2} = \int \frac{du}{(u+1)(u+2)}$$

$$\begin{aligned} \frac{1}{(u+1)(u+2)} &= \frac{A}{u+1} + \frac{B}{u+2} \\ 1 &= A(u+2) + B(u+1) \end{aligned}$$

$$\text{For } u = -2, \quad B = -1$$

$$\text{For } u = -1, \quad A = 1$$

$$\begin{aligned} \text{So, } \int \frac{du}{(u+1)(u+2)} &= \int \frac{du}{u+1} - \int \frac{du}{u+2} \\ &= \ln|u+1| - \ln|u+2| + C \\ &= \ln|\tan t + 1| - \ln|\tan t + 2| + C \end{aligned}$$