Math 2423-200 Quiz 1 Summer 2018

Name: Solution

Student Number:

Given the sum formulas we used in class, in case you need them:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

Problem 1

Evaluate $\int_0^2 (1+x^2)\,dx$ using the definition of the definite integral as the limit of a Riemann sum.

$f(x) = 1 + x^2$, $a = 0$, $b = 2$

(1) $\Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$

(2) $x_i = a + i\Delta x = 0 + i\frac{2}{n} = i\frac{2}{n}$

(3) $f(x_i) = 1 + x_i^2 = 1 + \frac{4}{n^2} \cdot i^2$

(4) $\sum_{i=1}^{n} f(x_i) \cdot \Delta x = \sum_{i=1}^{n} \left(1 + \frac{4}{n^2} i^2\right) \cdot \frac{2}{n} = \frac{2}{n}\left(\sum_{i=1}^{n} 1 + \frac{4}{n^2} i^2\right)$

$= \frac{2}{n}\left[\sum_{i=1}^{n} 1 + \sum_{i=1}^{n} \frac{4}{n^2} i^2\right]$

$= \frac{2}{n}\left[n + \frac{4}{n^2} \cdot \sum_{i=1}^{n} i^2\right]$

$= 2 + \frac{8}{n^3} \frac{n(n+1)(2n+1)}{6}$

(5) $\lim_{n\to\infty} R_n = \lim_{n\to\infty}\left[2 + \frac{8}{6}\,\frac{n+1}{n} \cdot \frac{2n+1}{n}\right] = 2 + \frac{8}{6}(1)(2)$

$= 2 + \frac{8}{3}$

Problem 2

Express the limit as a definite integral on the given interval. No explanation needed!

$$\lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{x_i\sqrt{1+x_i^3}}_{f(x_i)}\,\Delta x, \quad [2,5].$$

Compare: $\lim_{n\to\infty} \sum_{i=1}^{n} (x_i\sqrt{1+x_i^3})\,\Delta x = \underbrace{\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x}_{\int_a^b f(x)\,dx}$

So, $\lim_{n\to\infty} \sum_{i=1}^{n} x_i\sqrt{1+x_i^3}\,\Delta x = \int_2^5 x\sqrt{(1+x^3)}\,dx$