

Name: Solution

Student Number:

Problem 1

Find the derivative of the function

$$F(x) = \int_{\sin x}^{\pi/2} \frac{t}{\sqrt{1+t^2}} dt$$

We're going to use FTC I:

$$F(x) = \int_{\sin x}^{\pi/2} \frac{t}{\sqrt{1+t^2}} dt = - \int_{\pi/2}^{\sin x} \frac{t}{\sqrt{1+t^2}} dt$$

Since $f(t) = \frac{t}{\sqrt{1+t^2}}$ is continuous on \mathbb{R} , then

$$\begin{aligned} F'(x) &= - \frac{\sin x}{1 + (\sin x)^2} \cdot (\sin x)' \\ &= - \frac{\sin x}{1 + (\sin x)^2} \cdot \cos x \end{aligned}$$

Note: Some of you might have tried to evaluate the integral first, which is fine but remember, you need to differentiate your result! This is then a longer way!

Problem 2

Evaluate the integral

$$\int_1^2 (4x^3 - 3x^2 + 2x) dx$$

$$\begin{aligned} &\int_1^2 (4x^3 - 3x^2 + 2x) dx \\ &= \left(4 \cdot \frac{x^4}{4} - 3 \frac{x^3}{3} + 2 \frac{x^2}{2} \right) \Big|_1^2 \\ &= (x^4 - x^3 + x^2) \Big|_1^2 = (16 - 8 + 4) - (1 - 1 + 1) \\ &= 12 - 1 = 11 \end{aligned}$$