Math 2423-200 Quiz 4 Summer 2018

Name: Solution

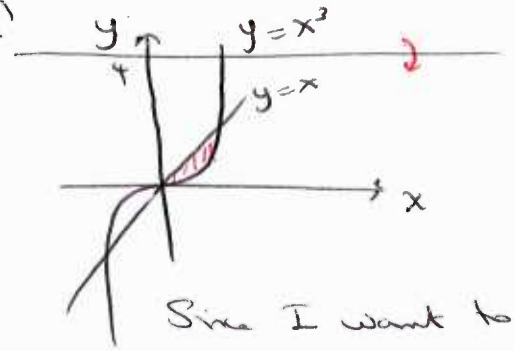
Student Number:

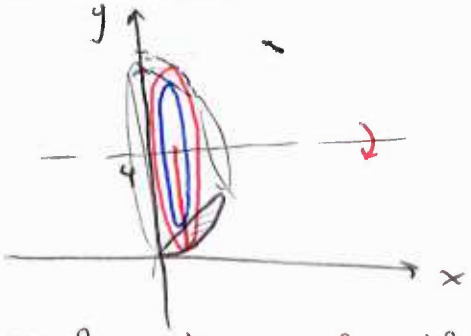
Problem 1

Let R be the region enclosed by the curves $y = x^3$, $y = x$ where $x \geq 0$.

(a) Let S be the solid obtained by rotating the region R about the line $y = 4$. Write an integral which has x as a variable and which computes the volume of the solid S.
You do not have to evaluate the integral.

Step 1





Since I want to set up an integral in terms of x, then I should use washers (thickness is Δx)

Step 2

$r_{out} = 4 - x^3$

$r_{in} = 4 - x$

$A(x) = \pi(4-x^3)^2 - \pi(4-x)^2$

Step 3: Bounds: $x = 0$ & $x = 1$

Step 4:

$$V = \int_0^1 \left[\pi(4-x^3)^2 - \pi(4-x)^2\right] dx$$

(b) Let T be the solid obtained by rotating the region R about the line $x = 7$. Write an integral that has x as a variable and which computes the volume of the solid T.
You do not have to evaluate the integral.

Since I need the integral in terms of x, if I use washers then my thickness is going to be Δy. Instead, use volume of cylindrical shells:

Thickness $= \Delta x$

radius $= 7 - x$

height $= y_T - y_B = x - x^3$

$$V = \int_0^1 \underbrace{2\pi(7-x)}_{\text{circumference}} \cdot \underbrace{(x-x^3)}_{\text{height}} \underbrace{dx}_{\text{thickness}}$$