

Name: *Solution*

Student Number:

Problem 1

Find the average value of the function over the given interval.

$$f(t) = \frac{t^2}{(1+t^3)^4}, \quad [0, 2]$$

$$f_{\text{ave.}} = \frac{1}{2-0} \int_0^2 \frac{t^2}{(1+t^3)^4} dt$$

$$u = 1+t^3 \quad t=0 \rightarrow u=1$$

$$du = 3t^2 dt \quad t=2 \rightarrow u=9$$

$$f_{\text{ave.}} = \frac{1}{2} \int_0^2 \frac{t^2}{(1+t^3)^4} dt = \frac{1}{2} \cdot \frac{1}{3} \int_1^9 \frac{du}{u^4}$$

$$= \frac{1}{6} \left[\frac{u^{-3}}{-3} \right]_1^9 = -\frac{1}{18} \left[\frac{1}{u^3} \right]_1^9$$

$$= -\frac{1}{18} \left[\frac{1}{9^3} - 1 \right]$$

Problem 2

If $f(x) = \frac{4x-1}{2x+3}$, find $f^{-1}(3)$.To find $f^{-1}(3)$, we need to find x s.t. $f(x) = 3$.

Solve: $f(x) = 3$

$$\frac{4x-1}{2x+3} = 3$$

$$4x-1 = 3(2x+3)$$

$$4x-1 = 6x+9$$

$$2x+10 = 0$$

$$x = -5$$

So, $f^{-1}(3) = -5$.