

Name: *Solution*

Student Number:

## Problem 1

Evaluate the following integrals.

(a)  $\int_0^1 (x^e + e^x) dx$

$$\begin{aligned} \int_0^1 (x^e + e^x) dx &= \left[ \frac{x^{e+1}}{e+1} + e^x \right]_0^1 \\ &= \left( \frac{1}{e+1} + e \right) - (0 + 1) \\ &= \frac{1}{e+1} + e - 1 \end{aligned}$$

(b)  $\int_0^{1/4} \frac{dx}{1+16x^2} = \int_0^{1/4} \frac{dx}{1+(4x)^2}$

$$\begin{aligned} \text{let } u &= 4x \\ du &= 4 dx \\ \frac{1}{4} du &= dx \end{aligned}$$

$$\begin{aligned} x=0 &\rightarrow u=0 \\ x=\frac{1}{4} &\rightarrow u=1 \end{aligned}$$

$$\begin{aligned} \int_0^{1/4} \frac{dx}{1+16x^2} &= \int_0^1 \frac{\frac{1}{4} du}{1+u^2} = \frac{1}{4} \cdot \tan^{-1}(u) \Big|_0^1 \\ &= \frac{1}{4} \cdot [\tan^{-1}(1) - \tan^{-1}(0)] \\ &= \frac{1}{4} \cdot \left[ \frac{\pi}{4} - 0 \right] = \frac{\pi}{16} \end{aligned}$$