Math 2423-200 Quiz 9 Summer 2018

Name: Solution

Student Number:

Problem 1
Evaluate the integrals.

(a) $\int \sin^3\theta \cos^4\theta \, d\theta$

$$= \int \sin\theta \cdot \sin^2\theta \cos^4\theta \, d\theta$$

$$= \int \sin\theta \cdot (1-\cos^2\theta)\cos^4\theta \, d\theta$$

let $u = \cos\theta$

$du = -\sin\theta \, d\theta$

$$\int \sin^3\theta \cos^4\theta \, d\theta = \int (1-u^2)\, u^4 \cdot (-du)$$

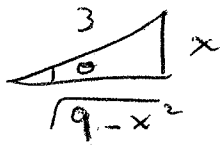
$$= -\int (u^4 - u^6)\, du$$

$$= -\left(\frac{u^5}{5} - \frac{u^7}{7}\right) + C$$

$$= -\left(\frac{\cos^5\theta}{5} - \frac{\cos^7\theta}{7}\right) + C$$

(b) $\int \frac{x^2}{\sqrt{9-x^2}}\, dx$

$$= \int \frac{x^2}{\sqrt{9\left(1-\frac{x^2}{9}\right)}}\, dx = \frac{1}{3}\int \frac{x^2}{\sqrt{1-\left(\frac{x}{3}\right)^2}}\, dx$$



let $\frac{x}{3} = \sin\theta \longrightarrow x = 3\sin\theta$

$\frac{1}{3}\, dx = \cos\theta \, d\theta$

$$\int \frac{x^2}{\sqrt{9-x^2}}\, dx = \frac{1}{3}\int \frac{(3\sin\theta)^2}{\cos\theta}\, 3\cos\theta \, d\theta$$

$$= 9\int \sin^2\theta \, d\theta$$

$$= 9\int \frac{1-\cos 2\theta}{2}\, d\theta$$

$$= 9\left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right) + C$$

$$= 9\left(\frac{1}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{1}{2}\,\frac{x}{3}\cdot\frac{\sqrt{9-x^2}}{3}\right) + C$$

Note:

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin\theta = \frac{x}{3}$

$\cos\theta = \frac{\sqrt{9-x^2}}{3}$

$\theta = \sin^{-1}\left(\frac{x}{3}\right)$