Math 4400 Homework 3 Hints

Problem 5 To figure out how to deal with that power of 100, try taking different numbers and seeing if there's a pattern to their powers mod 5. For instance, if we start with the number 2, its powers mod 5 are:

$$2^1 \cong 2,\ 2^2 \cong 4,\ 2^3 \cong 3,\ 2^4 \cong 2^3 \cdot 2 \cong 3 \cdot 2 \cong 1,\ 2^4 \cong 1 \cdot 2 \cong 2, \ldots$$

Problem 6 $a^2 - 3$ is divisible by 4 if and only if $a^2 - 3 \cong 0$ mod 4.

Problem 7 For this problem, it's necessary to understand how to think of the digits of an integer in a precise way that you can use to do proofs. Here's how this works: for any integer n, the digits of n are $n_0, n_1, \ldots, n_d$ if for all i such that $0 \leq i \leq d$ we have:

- $n_i \in \mathbb{Z}$,
- $0 \leq n_i \leq 9$, and
- $n = \sum_{i=0}^{d} 10^{d-i} n_i$

E.g. the digits of 1024 are $n_0 = 1, n_1 = 0, n_2 = 2, n_3 = 4$, because

$$1024 = 10^3 \cdot 1 + 10^2 \cdot 0 + 10^1 \cdot 2 + 10^0 \cdot 4$$