Name: Solutions

# Math 4400 Quiz 4

June 26, 2016

Instructions: You have until the end of class to complete this quiz. This quiz is two pages, and worth 20 points. Make sure to write your name at the top of the quiz. Show all of your work for full credit!

1. (10 points) Compute the order of [18] in the group $\mathbb{Z}/21\mathbb{Z}$. (Here, as usual, we're considering $\mathbb{Z}/21\mathbb{Z}$ as a group under addition)

By a homework problem (7b on HW 5),

$$o([18]) = \frac{21}{\gcd(18,21)} = \frac{21}{3} = \underline{\underline{7}}.$$

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ALTERNATIVELY:

We can just add [18] to itself until we get [0]:

$1 \cdot [18] = [18]$

$2 \cdot [18] = [15]$

$3 \cdot [18] = [12]$

$4 \cdot [18] = [9]$

$5 \cdot [18] = [6]$

$6 \cdot [18] = [3]$

$7 \cdot [18] = [0] \longrightarrow$ So $o([18]) = 7$.

2. (10 points) Prove that the set of linear polynomials with integer coefficients, i.e. $\{ax + b \mid a, b \in \mathbb{Z}\}$, forms an *abelian* group under the usual addition rule:

$$(ax + b) + (cx + d) = (a + c)x + (b + d)$$

- If $a, b, c, d \in \mathbb{Z}$, then $a+c \in \mathbb{Z}$ and $b+d \in \mathbb{Z}$, so this is a binary operation.

- Id element is $0 = 0x + 0$; since
  $(0x+0) + (ax+b) = (ax+b) + (0x+0) = ax+b$
  for all $a, b \in \mathbb{Z}$

- Associativity: $\forall a, b, c, d, e, f \in \mathbb{Z}$:
  $[(ax+b) + (cx+d)] + (ex+f) = [(a+c)x + (b+d)] + (ex+f)$
  $= ((a+c)+e)x + ((b+d)+f)$
  $= [a+(c+e)]x + [b+(d+f)]$
  $= (ax+b) + (c+e)x + (d+f) = (ax+b) + [(cx+d) + (ex+f)]$ ✓

- Inverses: the inverse of $ax+b$ is $(-a)x + (-b)$:
  $ax+b + [-ax-b] = (-ax-b) + [ax+b] = 0$
  If $a, b \in \mathbb{Z}$, then $-a, -b \in \mathbb{Z}$, so $(-a)x + (-b)$ is in our set.

- Abelian property: $\forall a, b, c, d \in \mathbb{Z}$:
  $(ax+b) + (cx+d) = (a+c)x + (b+d) = (c+a)x + (d+b) = (cx+d) + (ax+b)$

✓