

Math 1220-003, Summer 2018

Final Exam

Please write your name on the front and back of the exam. Remember to turn off your phone before starting this exam. Show all of your work for full credit. You may not use any notes or calculators during this exam.

Name: _____

UID: _____

1. (10 points) True or false:

(a) If $\sum_{i=0}^{\infty} a_i$ converges, then $\lim_{i \rightarrow \infty} a_i = 0$

(b) If $\lim_{i \rightarrow \infty} a_i = 0$, then $\sum_{i=0}^{\infty} a_i$ converges.

(c) If the power series $\sum_{i=0}^{\infty} a_i(x-1)^i$ converges at $x = -1$, it converges at $x = 2$.

(d) If $\sum_{i=0}^{\infty} |a_i|$ converges, then $\sum_{i=0}^{\infty} a_i$ converges

(e) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{2\pi}{3}$

2. Find the following derivatives:

(a) (6 points) $\frac{d}{dx} \tan(e^{3x})$

(b) (6 points) $\frac{d}{dx} x^{\sin x}$

3. Find the following limits:

(a) (4 points) $\lim_{x \rightarrow 0^+} \frac{\sin^{-1} x}{x}$

(b) (5 points) $\lim_{x \rightarrow 0^+} \frac{1}{x} - \frac{1}{x^2}$

4. Evaluate the following integrals:

(a) (6 points) $\int \frac{1}{x^2 + 2x + 5} dx$

(b) (6 points) $\int \frac{x}{\sqrt{1-x^2}} dx$

(c) (6 points) $\int \sin^3(x) \cos^4(x) dx$

(d) (6 points) $\int x \sin x dx$

5. (15 points) For each of the following series, write “CONVERGE” if the series converges and “DIVERGE” if the series diverges. You do not have to show your work for this problem.

(a)
$$\sum_{n=0}^{\infty} \frac{(-1)^n}{3^n}$$

(b)
$$\sum_{n=0}^{\infty} \frac{n!}{(n+1)^2}$$

(c)
$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n^2}}$$

(d)
$$\sum_{n=1}^{\infty} \frac{n+2}{n+1}$$

(e)
$$\sum_{n=1}^{\infty} \frac{n^3 + n + 1}{n^5 + n^4 + n^3}$$

6. (6 points) Salt water, at a concentration of 3 kg/L, flows into a tank of water at a rate of 2 L/min. Salt water flows out of the tank at a rate of 7 L/min. The tank starts with 10 Liters of water and 1 kilogram of salt. Find the differential equation describing the amount of salt in the tank after t minutes. (You don't have to solve this differential equation).

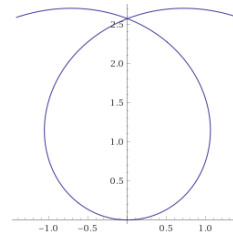
7. (6 points) Find the first three terms of the Taylor series of $f(x) = \frac{1}{\sin x}$ at $x = \frac{\pi}{2}$.

8. (6 points) Find the convergence set of the power series $\sum_{n=0}^{\infty} \frac{x^n}{n2^n}$.

9. (6 points) Match the equation in polar coordinates to the graph.

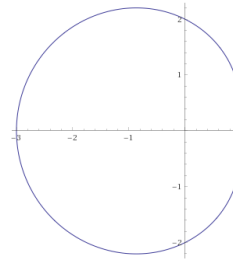
(a) $r = \sin(3\theta)$

1.



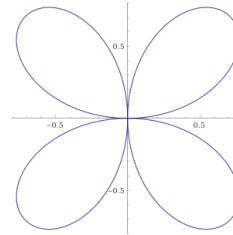
(b) $r = \sin(2\theta)$

2.



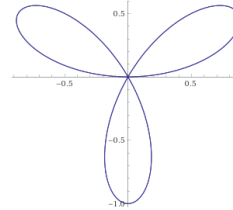
(c) $r = 2 - \cos \theta$

3.



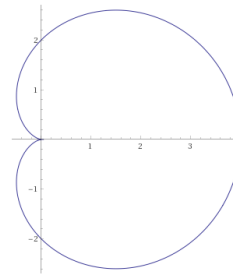
(d) $r = 2 + 2 \cos \theta$

4.



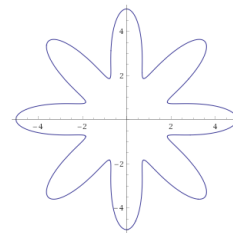
(e) $r = 2 + 3 \cos^2(4\theta)$

5.



(f) $r = \theta + \sin(\theta)$

6.



10. (6 points) Find the area bounded by the polar curve $r = \cos(3\theta)$, $-\frac{\pi}{6} \leq \theta \leq \frac{\pi}{6}$.

Name: _____

Page	Points	Score
2	10	
3	12	
4	9	
5	12	
6	12	
7	15	
8	6	
9	12	
10	6	
11	6	
Total:	100	